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Photographic Granularity and Graininess*

III. Some Characteristics of the Visual System of Importance in the Evaluation of Graininess and Granularity

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The literature concerning the characteristics of the human visual system is reviewed, with special emphasis on the dioptrical and histological properties of the eye and its closely associated nerve structures which are most relevant to the perception of graininess. From the histological evidence and from the results of measurements on visual acuity reported in this paper and in the literature, the conclusion is drawn that the neural responses from relatively few cones situated around the center of the fovea centralis are determinants of the perception of graininess. When viewing a granular field, these cones are stimulated by a rapidly changing illuminance, since, even during attempted fixation on a point in the visual field, the eyeball is not absolutely stationary but is subject to a multidirectional vibratory motion of relatively small amplitudes and high frequencies, the physiological nystagmus. Microdensitometric traces on a group of developed photographic deposits were obtained which show the characteristics of the stimuli incident upon the retinal receptors which are supraliminal, liminal, and subliminal with respect to graininess perception. Since the magnitude of the neural response of about 80 percent of the foveal cones is determined largely by the temporal rate of change in the stimulus incident thereon, it appears that the most relevant characteristics of the stimuli mentioned in the previous sentence are the temporal gradients (that is, the time rate of change) of the stimuli. Merging distance measurements were made on several types of graininess patterns of known geometric design, and from these the spatial distributions of illuminance on the fovea corresponding to the graininess threshold were computed. The temporal illuminance gradients on the retinal receptors when these conventionalized patterns were viewed at the blending distance were also computed by employing the most probable values for the constants of the physiological nystagmic motion of the eye. From these computations it is apparent that the illuminance gradient required to evoke the perception of threshold graininess decreases as the average transmittance of the sample (and hence the average illuminance incident on the retina) decreases. Hence the sensitivity of a receptor to the time rate of change in the incident illuminance varies inversely with the average illuminance to which it is adapted. This functional relation is defined as the *gradient sensitivity* of a retinal receptor. In the measurement of graininess, either by the variable viewing distance or by the variable magnification techniques, the average illuminance incident on the retina (the adapting illuminance) decreases as the density of the developed silver deposit increases. Hence the gradient sensitivity of the receptors increases as the density of the silver deposit increases. At the same time the configuration of the pattern in the visual field, which is determined by the number, size, shape, and distribution of the developed grains and clumps thereof (and possibly the luminous transmittance of these visual field elements) changes as the density of the silver deposit increases from a low to a high value. These purely objective characteristics of the developed silver image determine that component of the total graininess which may be termed the *structural component* of graininess. These same structural aspects of the silver deposit also determine the *granularity* of the sample. Granularity may be measured in a large number of different ways, each leading to a unique numerical value. Each of these values can be defined in terms of the operations

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used in its determination. Since we are seeking a method of measuring granularity which will yield values which correlate either with the total graininess or with the structural component of graininess of a sample or a group of samples, it is evident that the desired operational definition of granularity must be in harmony with the modes of functioning of the visual system. From an analysis of these modes, it is concluded that the most promising procedure is to evaluate granularity in terms of the frequency of occurrence of the density or transmittance differences existing between a large number of pairs of surface elements of the sample, the two members of each pair of elements being of *equal area* and immediately *adjacent* to each other. Such density (or transmittance) differences are termed *syzygetic density* (or transmittance) *differences*, $S\Delta D$, or $S\Delta T$. These particular types of differences must not be confused with the density (or transmittance) differences, ΔD or ΔT, obtained by using the *average transmittance*, $\bar{T}$, or the *integrated density*, ID, of the whole sample as a basis for computation. It is probable that not *all* of the $S\Delta D$ or the $S\Delta T$ values are relevant for the evaluation of granularity, but that those values less than some limit, x, and greater than another limit, y, should be discarded. Moreover, the size of the surface element used in the determination of $S\Delta D$ or $S\Delta T$ should be related in some definite manner to the size of the central foveal cones. The evidence indicates that the most probable relationship is that the surface elements of the sample should have a size very nearly equal to that of central retinal cones as projected by the dioptrical elements of the eye onto the sample when viewed at a distance such that the graininess is just perceptible. Since the photographic materials in which we are interested vary over a wide range with respect to the coarseness of their granular structures, it is extremely unlikely that the required values of $S\Delta D$ or $S\Delta T$ for the whole gamut of materials can be obtained by employing scanning apertures of fixed sizes in the microprojector used for their measurement. Hence an instrumental technique involving *constant* magnification with scanning apertures of *variable* size, or a *variable* magnification with scanning apertures of *constant* size, is indicated. The problem of establishing a graininess scale having uniform subjective intervals which involves the determination of *just noticeable graininess differences* is discussed. New instruments, now under construction, having characteristics in conformity with the requirements based upon conclusions reached by a study of the modes of functioning of the visual system are discussed briefly.

I. INTRODUCTION

IT has been shown in a previous communication[1] that the value of granularity depends, among other factors, upon the size of the scanning aperture which is used in the measurement of the density or transmittance variations. Some semiquantitative data, in the form of composite photomicrographs (Figs. 27 and 28) of granular structures, were given showing the relation between the size of the foveal receptors and the visual field elements at the threshold of just-perceptible graininess. These graphic data indicate that the retinal image incident upon each individual foveal receptor (cone) consists of a relatively small number of visual field elements, one or two to not more than eight or ten. The average number of visual field elements imaged on each receptor is approximately the same for four different photographic materials differing widely with respect to grain size, granularity, and graininess. Other photomicrographs were given showing the structural characteristics of developed photographic silver deposits isolated, in the measurement of granularity, by the use of scanning apertures differing in size. These indicate that little correlation can be expected between measurements of granularity and of graininess unless the size of the scanning aperture is reduced to a point where the number of grains or clumps thereof is diminished to a relatively low value.

If granularity values are to serve as significant indications of the visual appearance (that is, graininess) of the silver deposits, we should expect to find some definite and relevant relationship between the size of the scanning aperture and the effective size of the sensitive receptors in the retina of the observer's eye. These receptors are of two kinds, namely, rods and cones. In that region of the retina, the fovea centralis, which receives an image of an object on which attention is fixed, no rods are present and the cones are smaller in cross section than in the extra-foveal regions of the retina. It seems probable that the cross-sectional size of the cones in the fovea, or perhaps we should say the spacing or distance, CD, between the centers of adjacent cones, is one of the determinants of the minimum size of an object or of the least distance between objects which, at a given

[1] L. A. Jones and G. C. Higgins, "Photographic granularity and graininess. II. The effects of variations in instrumental and analytical techniques," J. Opt. Soc. Am. **36**, 203 (1946).

TABLE I. Published values of focal lengths of the human eye.

	Far		Near	
	F_1	F_2	F_1	F_2
Helmholtz	15.0072	20.0746		
Helmholtz	14.858	19.875	13.274	17.756
Tscherning	17.1	22.9		
Gullstrand	17.055	22.785	14.169	18.930
Polyak	17.18			
Moon			15.0	
Byram			16.0	
Hecht			16.5*	

* Distance of test object "between 2 and 3 meters."

viewing distance and under a specified set of conditions, can be perceived.

When a developed silver image is viewed with suitable magnification, or in some cases even without magnification, it appears as a two-dimensional manifold made up of a complex pattern or configuration consisting of a multitude of elements varying in size, shape, grouping, and luminance. Just as in the case of simple test objects, such as are commonly used in the evaluation of the resolving power or visual acuity of the eye, the size of the retinal receptors should be one of the determinants of the magnification or viewing distance at which the structural detail of the silver deposit becomes just imperceptible.

II. SOME CHARACTERISTICS OF THE VISUAL SYSTEM

An examination of the literature concerned with the histological and dioptrical characteristics of the human eye indicates that there is poor agreement between the quantitative values relating to the structure of the visual organ. For our present purpose it is unnecessary to consider in detail all of these optical and anatomical characteristics. We are chiefly interested in the size of the retinal receptors and in the focal lengths of the optical system. We make no pretense of having made a complete and critical study of the voluminous literature. We shall, however, quote some values in the following paragraphs to illustrate the variability of the information on this subject and to aid us in choosing reasonable values to use in subsequent computations.

Focal Lengths of the Average Human Eye

In seeking information on this subject it was natural to turn first to the classical work of Helmholtz,[2] the first complete edition of which was published in 1867. In this we find (on pages 68 and 111) complete data on the dioptrical characteristics of the human eye, relaxed (for distant vision) and accommodated for near objects. In the first line of Table I are shown values (from page 68, reference 2) for the first, F_1, and the second, F_2, focal lengths for distant vision. According to Helmholtz, these values are based upon Listing's[3] schematic eye, concerning which he says:

> There still remains to be stated what has been learned to date about the cardinal points of the eye. The conclusions here given are based on Listing's schematic eye, which certainly departs very little from the actual average, as is verified in some measure by the writer's investigations. At any rate, wherever it is necessary or permissible to use mean values in the computations of physiological optics, the numerical data for the individual eye in question being unknown, it is probable, in view of the very large individual variations that exist, that the data of Listing's schematic eye will be found just as reliable and satisfactory as the actual mean values, supposing that the latter were known. Accordingly, Listing's constants will be used in the treatise, with occasional comments, when necessary, explaining how they are sometimes different from what seem to be the actual mean values.

On page 111 of the same treatise is found a second table giving the characteristics of a schematic eye which includes the values of focal lengths when accommodated for both near and far vision. These are given in the second line of Table I.

In 1892 Tscherning[4] published values obtained by careful measurement on a single specimen. The values are shown in the third line of Table I.

Subsequent to the death of Helmholtz, the third German edition[5] (1909) of his treatise appeared, in which we find values agreeing with those of the first edition (1867), as shown in the first and second lines of Table I. This third German edition contained, in addition to the text of the first edition, extensive appendices by

[2] H. Helmholtz, *Handbuch der physiologischen Optik* (L. Voss, Leipzig, 1867).
[3] J. B. Listing, "Dioptrik des Auges", R. Wagners Handwörterbuch der Physiologie **4**, 451 (1851).
[4] T. H. Tscherning, *Optique physiologique* (Paris, 1898).
[5] H. Helmholtz, *Handbuch der Physiologischen Optik*, third German edition (1909).

Gullstrand,[6] in one of which are given the characteristics of a schematic eye as established by him. The values from this source are shown in the fourth line of Table I. The near-vision values are for an object 130.09 mm in front of the vertex of the cornea. We assume that these values are based on more modern and more voluminous experimental measurements which became available between the time of Listing's work (1851) and the publication of Gullstrand's appendix to the third edition of Helmholtz's work (1909). We further assume that in this case also the values represent an average normal human eye.

In 1924 an English translation[7] of the third German edition of Helmholtz's treatise appeared. In this may be found the values based upon Listing's schematic eye (page 152) and those given by Gullstrand (pages 351 and 392).

In a recent comprehensive monograph by Polyak[8] on the structure of the retina, some data are given concerning the size of localized retinal areas and their corresponding angular subtenses in the visual field. From these we have computed the value of F_1 which is necessary to make Polyak's values concordant. We assume that these angular values refer to the relaxed eye, that is, distant vision. This value of F_1 is shown in the fifth line of Table I (see also Table II).

Moon,[9] in his discussion of visual acuity and the resolving power of the human eye, uses a value of 15 mm for F_1 and states that the retinal cones (presumably in the fovea) are "small in size (approximately 3.2μ), are packed tightly together, and assume a hexagonal shape." He says nothing concerning the condition of accommodation to which the focal length of 15 mm applies, but if this is for the condition of test objects relatively close to the observer, the value is only a little greater than that given by Gullstrand for near vision.

In a recent publication by Byram[10] (page 723) a value of 16 mm for F_1 is used in computing the resolving power of the retina. Again, no statement is made concerning the accommodation condition to which this value applies, but if it applies to observations made on a test object relatively close to the observer, his value is about 2 mm greater than that of Gullstrand's for the near condition of accommodation.

Hecht,[11] in a report on the visibility of wires, gives values of image width and angular subtense which require the use of a value for F_1 of 16.55 which is in excellent agreement with Gullstrand's values when corrected for an object distance of 250 cm, approximately that used by Hecht. Incidentally, his statement that a width of 0.04μ on the retina is about 1/65th the retinal cone width, leads to a cone diameter value of 2.60μ.

Since we shall find it expedient to refer frequently to Polyak's work and his values for the various angular subtenses corresponding to linear dimensions on the retinal surface in the foveal region, we shall use his value of 17.18 for F_1 in order that these magnitudes shall be concordant. Moreover, 17.18 agrees reasonably well with the values given by Tscherning and Gullstrand. Throughout this paper, therefore, where it is necessary to use a value of F_1 (for instance, in computing the angular subtense of a retinal receptor or of an element in the visual field), we shall use 17.18 mm.

Cone Spacing in the Fovea

The data concerning the cross-sectional size of the retinal receptors are also discordant, although some of the disagreement doubtless may be attributed to the actual differences in size of the cones in various parts of the retina. In the first edition of Helmholtz' work (see page 19, reference 2) we find the following statement: "The cones are found between the rods. They are thicker (from 0.0045 to 0.0065 mm) and shorter than the rods and are com-

[6] A. Gullstrand, Appendix to Helmholtz' third German edition (1909).

[7] H. Helmholtz' treatise on physiological optics, translated from third German edition, edited by J. P. C. Southall, published by Optical Society of America (G. Banta Company, Menasha, Wisconsin, 1924–25).

[8] S. L. Polyak, *The Retina* (University of Chicago Press, Chicago, 1941).

[9] P. Moon, *Scientific Basis of Illuminating Engineering* (McGraw-Hill Book Company, Inc., New York, 1936), p. 423.

[10] G. M. Byram, "The physical and photochemical basis of visual resolving power. I. The distribution of illumination in retinal images," J. Opt. Soc. Am. **34**, 571 (1944); "II. Visual acuity and the photochemistry of the retina," J. Opt. Soc. Am. **34**, 718 (1944).

[11] S. Hecht and E. U. Mintz, "Intensity and the visibility of wires," J. Opt. Soc. Am. **28**, 177 (1938).

posed of a similar substance." These values obviously apply to cones outside of the rod-free area. In the same publication are given some values reported by other investigators (page 22, reference 2):

Diameter of Cones

A. Kölliker:—0.0045 to 0.0067.
M. di Vintschgau:—0.0034 to 0.0068.

In the Yellow Spot

A. Kölliker:—0.0045 to 0.0054.

Concerning the first two of these values, no statement is available as to the locality of the retina to which these measurements apply. In the case of the last reference, it is stated that the measurement was made some place in the yellow spot (macula lutea). However, according to Polyak, the yellow spot is about 3 mm in diameter centered around the visual axis, while the rod-free area has a diameter of only about 500μ. These values by Kölliker, therefore, may not refer to a cone in the rod-free area.

In a supplement (from the first edition of Helmholtz' treatise, page 822, *et seq.*) some further detailed information is given concerning the cone diameter:

> The diameter of the thicker, inner part of the cones gets to be as much as from 0.004 to 0.006 mm. Only in the fovea where there are no rods between the cones, the latter are thinner. Their basal ends measure from 0.002 to 0.0025 mm, according to M. Schultze, and in a small region from 0.0015 to 0.002 mm, according to H. Müller, and between 0.0031 and 0.0036 mm, according to Welcker.

In addition to the values attributed to Schultze in this quotation, he has reported[12] also that at the center of the fovea the long slender cones were found to be very crowded together and were only 0.00066 mm (0.66μ) across.

Polyak[8] gives numerous data concerning the dimensions of the cones. For convenience in discussing the histological characteristics of various parts of the retina, he has divided the entire retinal area into seven regions which are described as roughly circular and concentric. At present we are most concerned with that area which he calls the "central area" and its three subdivisions, namely: the "central fovea," the "parafoveal region," and the "perifoveal region." Concerning the distribution of the cones in the central fovea, Polyak states:

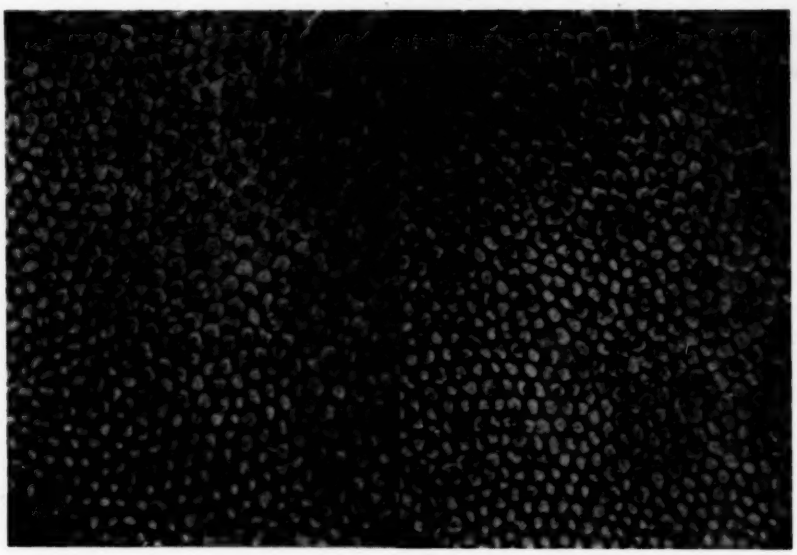
> Viewed in flat preparations, fresh as well as fixed and stained, the cones appear as a perfectly evenly distributed mosaic without any gaps except the narrow slits filled with the extremely thin neuroglial sheaths that separate adjoining cones from each other (Fig. 40). The thickness of the separating sheaths barely reached 0.3μ. This observation is fully substantiated by examination of whole-mount preparations of perfectly fresh retina taken from the eyeball immediately after enucleation. Such preparations show the cones arranged in even, somewhat arching, and crossing rows. In general, the arrangement or distribution of the central cones is distinguished by its exquisite regularity and uniformity.

The "Fig. 40" referred to in this quotation consists in part of two very excellent photomicrographs showing the shape and arrangement of the cones at two localities within the central fovea. These are reproduced, with the permission of the publisher and of the author, as Fig. 1 of this communication. Neither of these photographs represents the exact center of the central fovea. In Fig. 1 (b) it will be seen that the cones are somewhat smaller in cross section than those of 1 (a), the former representing a locality somewhat closer to the exact center of the fovea than the latter. Both close packing of the photoreceptors and the tendency of these retinal elements to assume hexagonal shapes are clearly evident from these photomicrographs. Polyak goes on to say that in his opinion the reports of irregular distributions of cones in the central area which have appeared from time to time are due to faulty techniques which result in unequal shrinking during the preparation of the sample.

A considerable portion of Polyak's data concerning the size of the foveal cones is contained in the following quotation, which gives the sizes in the three regions of the central area and the number of cones found in each:

> In a whole-mount preparation of a human specimen 25,000 cones were found in the territory of the outer fovea. This territory is almost coextensive with the foveola, where all inner layers below 5-*a* and all blood vessels are practically absent. In this locality, necessarily, the rays of light reach the photoreceptive layer with least obstruction. It therefore may well be the instrument of most discriminating or analytical vision or space perception (so-called "direct vision," "epicritic vision," incorrectly

[12] M. Schultze, "Ueber Stäbchen und Zapfen der Retina," Arch. f. Mikrosk. Anat. **3**, 215 (1867).



(a) (b)

FIG. 1. Photomicrographs showing the retinal mosaic of cones at two points near the center of the fovea. (*From Polyak.*)

also "foveal vision," etc.). It corresponds approximately to 1° 20′ of arc in the field of vision (40′ of arc on each side from the ideal fixation point). In Man this territory measures 400μ across and is practically homologous with the flat floor of the simian fovea. A smaller central island, measuring not more than 100μ across, where the cones are thinnest and practically uniform, corresponding with a minute central area of only 20′ of arc in the visual field (10′ on each side of the ideal fixation-point), would yield even a higher degree of visual acuity, since the number of cones here is 2500.

The human central territory, where the rods are absent, measures approximately 500μ or slightly more across (1° 40′, or 50′ of arc on both sides of the ideal fixation-point). The number of cones computed to this territory is approximately 34,000.

For convenience, data from this quotation are summarized in Table II, in which the numbers appearing in bold-faced type are Polyak's. From the angular extent, $\angle$ and $\frac{1}{2}\angle$, of the three foveal regions described and their measured diameters, ϕ_d, it is possible to compute F_1. These values are shown in the fifth column and indicate that Polyak used for F_1 a value of 17.18 mm. The number of cones in each foveal region is shown in the sixth column. If we assume further that the three regions are circular, and that in them the cones are so closely packed that they become hexagonal in cross section, it is possible to compute the average area occupied by each cone, A_c, and the average distance between adjacent cone centers, *CD*. These values are shown in the seventh and eighth columns of Table II. It will be seen that the distance between the centers, *CD*, is smallest (1.9μ) in the region described by Polyak as "a small central island" which occupies the center of the fovea centralis. The center distance increases to 2.41 for a retinal region having a diameter of 400; and to 2.59 if the entire rod-free area is considered.

The values of *CD* in Table II represent averages for the three regions to which they apply. It should not be concluded that the cone diameters are of uniform magnitude in each of these three regions. In discussing the characteristics of the fovea in greater detail, Polyak (page 425) points out that:

> The cone population, as already described, varies in size and in numbers per square unit of retinal surface from the foveal center to the periphery at the ora serrata. The thinnest cones are present in the very center of the fovea,

where their thickness, in Man, is reduced to almost 1μ, corresponding to a visual angle of approximately 12″ to 15″ of arc.

Even in the region *A* the cone diameters or their separations, *CD*, are not uniform, but those immediately around the center are smallest, subtending only about 12 seconds of arc. However, there are relatively few, perhaps only one or two dozen cones having this minimum size, which includes the insulating layers of neuroglial tissue separating the cones from each other. The angular subtense mentioned, therefore, applies to the center distance rather than to the actual cone diameter. As the distance from the center of the fovea increases, the cone size increases, this increase being sufficient to give an average value for the region *C* (Table II) which includes the entire territory in which there are no rods of 2.59μ.

Other workers in this field have used various values for the cone diameter. The majority of such values seem to fall between 2.0 and 3.2μ, but in most cases no statement is made as to the position in the retina, relative to foveal center, to which the quoted values of *CD* (or ϕ) apply. The values of diameter or of center distance computed from Polyak's data agree quite well in a general way with those given by Helmholtz[5] and by Schultze.[12]

For our purpose we desire more detailed information concerning the rate at which the center distance, *CD*, between adjacent cones increases outward from the visual axis. In Fig. 99 of reference 8 is given a curve showing the relationship between *CD* and the angular departure from the "ideal foveal center," by which we assume is meant the point at which the visual axis intersects the retinal surface. However, the abscissa scale is so compressed that it is difficult to read the most important values which, from our point of view, lie between 0 and 1 degree or less. If we take the data from Table II and other statements by Polyak, especially those given concerning the visual perception of space (pages 425 to 427), it is possible to establish, with a fair degree of certainty, the desired relationship in the region between 0 and 1 degree. This is shown in Fig. 2. The curve has been drawn in what seems to be its most probable position. At an angle of 5 minutes the value of *CD* as read from this curve is 1.5μ. The average value of *CD* for the cones within a circle of this radius must be somewhat less. The number of cones in this area (within a 5-minute radius), therefore, is of the order of 1200. From evidence given later in this paper it seems probable that the judgment of the disappearance of graininess is dependent upon the neural responses of a group of cones less numerous than the 1200 which inhabit a circle having a radius of 5 minutes (25μ on the retina) centered around the fixation point in the visual field or the center of the fovea.

In summary: The evidence, therefore, indicates that at the exact center of the fovea, where the visual axis meets the retina, the cones have a minimum diameter (or spacing) which may be as low as 0.66μ to 1.0μ. This increases progressively, the average within a circle having a diameter of 50μ being approximately 1.5μ (see Fig. 2), while the average for a central area 400μ in diameter rises to 2.4μ. The judgment of disappearance of detail in a visual field due, for example, to increase of the viewing distance, is probably conditioned largely by the characteristics of the retinal receptors lying relatively close to the visual axis. In making such a judgment, the observer usually deliberately and consciously directs his attention to various parts of the visual field and hence every point of the field is examined by that part of the retina lying at or immediately around the exact center of the central fovea, which, by definition, is the point where the visual axis intersects the retinal surface.

In subsequent computations requiring the use of a value of cone diameter, ϕ, or *CD*, we shall usually employ 1.5μ, although in some cases where the evidence indicates that an area of the fovea, immediately around the exact center,

TABLE II. Dimensions of foveal regions. *Definitions of the regions*—(*A*) The "smaller central island" within the territory, etc. (*B*) The "territory of the outer fovea." (*C*) The "human central territory, where the rods are absent."

Region	$\angle$	$\frac{1}{2}\angle$	θ_d	F_1	N	A_θ	CD
A	20′	10′	100μ	17.18 mm	2,500	3.14μ^2	1.91μ
B	80′	40′	400μ	17.18 mm	25,000	5.03μ^2	2.41μ
C	100′	50′	500μ	17.18 mm	34,000	5.78μ^2	2.59μ

having a radius less than 5 minutes (25μ) is functioning, it may be necessary to use a value of CD somewhat less than 1.5μ.

The Image on the Retina

Another important factor in the problem is the distribution of light in the image of an object formed on the retina by the refracting elements of the eye. This distribution is different from that which would be found in a perfect geometrical image. The spread of light outside of the geometrical limits of a bright object on a darker background or into the limits of the geometrical image of a dark object on a brighter background is caused largely by the diffraction of light by the pupillary aperture and to the spherical and chromatic aberration of the image-forming system of the eye. For a pupil diameter of approximately 2.5 mm or less, the spread is attributable almost entirely to diffraction, while for larger diameters both chromatic and spherical aberration may become of some importance. Scatter of light within the eye itself also must be considered since this tends to lower the illuminance contrast in the image on the retina. The dependence of visual acuity upon pupillary diameter has been investigated by Cobb.[13] His results show that acuity increases rapidly up to a diameter of 2 mm, remains practically constant between 2 and 4 mm, and then decreases slightly for greater diameters because of chromatic aberration.

The pioneer work on the problem of the distribution of diffracted light was done by Struve,[14] who derived a mathematical formula for the distributions of intensity in the image of a straight boundary line between a light and a dark area. A little later Rayleigh[15,16] published his classical work on the distribution of light in the image formed on the retina by the dioptrical elements of the human eye.

The distribution of illuminance[20] in the retinal image has been treated more or less exhaustively

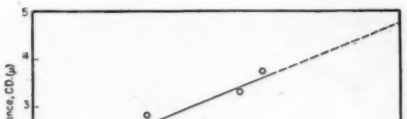


FIG. 2. Relation between cone diameter and angular separation from the center of the fovea.

by other workers in this field. Among the relatively recent contributions may be mentioned those of Hartridge,[17] Hecht and Mintz,[18] Moon,[9] Selwyn,[19] and Byram.[10] We shall not attempt a detailed review of this subject, but one simple case will be discussed for the sake of illustrating the general situation.

In Fig. 3 is shown the condition existing when the boundary, $M'N'$, between a light area (having a luminance of B_1) and a dark area (having a luminance of B_2), as illustrated at the top of the figure, is imaged on the retina. The geometrical image of the boundary is the line MN, and the corresponding distribution of the illuminance incident on the retina is given by the curve $lMONp$. Curve A shows the actual distri-

[13] P. W. Cobb, "The influence of pupillary diameter on visual acuity," Am. J. Physiol. **36**, 335 (1915).

[14] H. Struve, "Beitrag sur Theorie der Diffraction an Fernrohren," Ann. der Physik **17**, 1008 (1882).

[15] Lord Rayleigh, "Wave theory of light," Encyclopaedia Britannica, ninth edition, **24**, 421 (1888).

[16] Lord Rayleigh, "On the theory of optical images, with special reference to the microscope," J. Roy. Microscopical Soc. 447 (1903).

[17] H. Hartridge, "Visual acuity and the resolving power of the eye," J. Physiol. **57**, 52 (1922).

[18] S. Hecht and E. U. Mintz, "The visibility of single lines at various illuminations and the retinal basis of visual resolution," J. Gen. Physiol. **22**, 593 (1939).

[19] E. W. H. Selwyn, "Limit of visual resolution," Proc. Phys. Soc. (London) Part 4, **55**, 286 (1943).

[20] It is frequently necessary to distinguish clearly between the psychophysical concept, "luminance," and its subjective correlate, "brightness." We shall, therefore, in this paper use the terminology recommended by the Colorimetry Committee of the Optical Society of America [Colorimetry Report Opt. Soc. Am. "Psychophysics of color," J. Opt. Soc. Am. **34**, 245 (1944)], which includes among other things the replacement of the current term "illumination" by the term "illuminance," and the substitution of "luminance" for "brightness." "Brightness" is now defined as the subjective or psychological correlate of "luminance." The reader who is unfamiliar with the terminology, symbolism, and units recommended by the Colorimetry Committee of the OSA is urged to consult the reference given in order to familiarize himself with this material. Not only will this enable the reader to understand more fully the significance of this proposed terminology, but will give him a better understanding of the relationships between the fields of physics, psychophysics, and psychology, all of which are involved in this general treatment of the graininess and granularity of photographic materials.

bution due to diffraction, as computed by Struve's formula,* the pupillary diameter being 2.43 mm.**

We have chosen the case of a boundary separating a light and a dark area to illustrate the relation between the retinal image and the retinal mosaic when attention is fixed on a detail attributed to luminance differences in an object. The use of a boundary *line* reduces the problem to a one-dimensional one, and the concepts relating to it are easier to comprehend than the more complicated case of fixation on a *point* involving a two-dimensional distribution of the diffraction pattern on the retinal mosaic. When an object having a very small angular subtense (such as a star) is imaged on the retina, the diffraction pattern takes on a circular form and the falling off of illuminance along a radius of this circle is very similar (for our present purpose it may be taken as identical, see Fig. 1 of reference 10) in shape to that which exists perpendicular to a long straight boundary between a light and a dark area. Although we have been unable to find in the literature any quantitative data as to the physiological nystagmus which takes place when a point or a horizontal line is fixated, it seems reasonable to expect that the vertical components of the nystagmus should resemble quite closely, at least with respect to frequency and maximum amplitude, the motions reported by Adler and Fliegelman[21] which occurs when an attempt is made to fixate a vertical line. If the tensions on the muscles which control the horizontal movements of the eyeball are such as to cause the eye to vibrate according to its natural period, there seems to be no reason to doubt that the same or similar tensions exist in the muscles which control the vertical movement of the eyeball. It seems likely, therefore, that in the attempt to fixate a point the motions of the image on the retina should resemble in some respects at least those of a Lissajous figure resulting, for instance, from the use of two tuning forks having the same or only slightly different frequencies. At any rate, we should expect to find a multiplicity of directions in these high frequency tremors and frequencies similar to those which have been found in the case of fixation on a vertical line. Such motion should lead to rates of change of illuminance on the retinal receptors similar to those derived from Fig. 3.

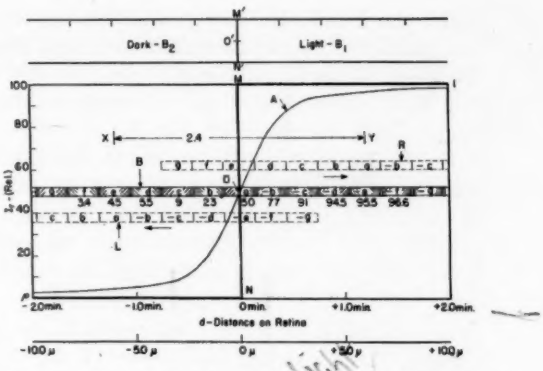


FIG. 3. The distribution of illuminance on the retina across the geometrical image of a boundary between a light and a dark area.

Let us assume, for the moment, a condition of absolute fixation, with the optical axis of the eye intersecting the object plane of the figure at *O'*, the fixation point in the lower part of the figure being at *O*. Furthermore, let us assume that the visual axis of the eye intersects the retinal surface at the center of a cone. We may draw in a hypothetical cone distribution as shown at *B* (Fig. 3). The cross-hatched areas represent the cones, and the spaces between represent the separating layers of neuroglial tissue which, according to Polyak, is not more than 0.2μ thick at the very center of the fovea, increasing to 0.3 in the outer zones of region *A* (Table II), and to 0.5μ or more at greater distances from the center. The diagram represents the condition at the exact center of the fovea. In this region, according to the best information available, the distance between cone centers is very small, lying between 0.66μ and 1.5μ. In the figure a cone center distance, *CD*, of 1.5μ is assumed. It seems probable that this value is somewhat greater than is actually the case. If we assume that the retinal image of the boundary line extends from a point where the illuminance is 4 percent less than that produced by the extended light area to the point

* This formula employs a Bessel function for obtaining the distribution of the diffracted light, not the Gaussian function which is erroneously used by some authors.

** Chromatic aberration for this diameter of pupil is small and for the present at least can be neglected.

[21] F. H. Adler and F. Fliegelman, "Influence of fixation on the visual acuity," Arch. f. Ophthalm. 12, 475 (1934).

where the illuminance is 4 percent greater than that produced by the extended dark area, then the width of this retinal image covers nine cones, that is, 4.5 cones on each side of the geometrical image of the boundary.

In Fig. 3 immediately below the cross-hatched area representing the cones are shown the values of the average relative retinal illuminance, I_r (neglecting entoptic scattered light), on each cone. On Cone b the average illuminance is 23 units compared to the average illuminance incident on Cone a of 50, the decrement being 54 percent. The percentage increase on Cone $-b$ is likewise 54 percent. We know that under favorable circumstances the eye can detect a difference of about 1 percent in the luminance of two relatively large juxtaposed areas, which means that for the static condition represented in Fig. 2 the illuminance on adjacent cones, a and b or a and $-b$, could differ only by about 0.25 percent. When we consider that the retinal receptors are subject to fatigue, it seems incredible that this condition, assuming a perfectly static state, could lead to the detection of a luminance difference between two parts of a field differing by only 1 percent.

The assumption of a larger cone diameter would change this picture somewhat. For instance, if it be assumed that the center distance between adjacent cones is three times the 1.5μ value used in the construction of Fig. 3, computations show that the difference between the illuminance incident on the central cone, a, and the adjacent cone, b (or $-b$), would be approximately 87 percent. This might be interpreted as indicating that a coarser retinal structure than that which we have assumed to exist at the center of the fovea would give a higher sensitivity to brightness differences. The experimental evidence indicates, however, that ΔB, the just noticeable luminance difference, increases at off-axis points, which still lie in the rod-free area (out to 50 minutes of arc) where CD is of the order of 4 or 5μ.

An examination of the literature shows clearly that great difficulties have been encountered in attempting to explain the observed facts of visual acuity, resolving power, and luminance discrimination on the basis of the static condition resulting from absolute fixation. Thus, Hecht,[22] in his discussion of luminance discrimination, recognizes the presence of "the usual and persistent eye movements which expose fresh portions of the retina to the higher intensity" when an observer adjusts a two-part visual field to show a just perceptible luminance difference. By recognizing the existence of this dynamic situation Hecht was able to formulate more satisfactorily theoretical explanations of the experimental facts of luminance discrimination (contrast sensitivity) than he could achieve by assuming the static condition demanded by perfect fixation. Marshall and Talbot[23] have put forth in a convincing manner a dynamic theory to account for the relation between retinal receptor sizes and the observed facts of visual acuity.

Moreover, practically all of the evidence indicates the impossibility of achieving absolute fixation. Even during the most strenuous attempts to fix the attention on a *point* or *line* the eye is in constant motion. Data concerning the precise nature of this motion are meager. However, Adler and Fliegelman[21] have obtained some quantitative data on the physiological nystagmus, which can be used to predict with a fair degree of certainty the way in which the eye moves during attempted fixation.

According to Adler and Fliegelman, this physiological nystagmus is a tremorlike motion of various frequencies and amplitudes. We must, therefore, picture the situation, represented in Fig. 3, as one in which an illuminance gradient is moving back and forth rapidly over the retinal mosaic lying in the immediate vicinity of the geometrical image of the boundary line, $M'N'$. Even if the evidence for the existence of relatively high frequency physiological nystagmus is not conclusive, it is still difficult to reconcile the facts of vision with a purely static situation. For instance, if we assume that the optical elements of the eye could possess such characteristics that a light source subtending a small angle could be imaged so as to illuminate only a single cone and

[22] S. Hecht, "A theory of visual intensity discrimination," J. Gen. Physiol. 18, 767 (1934–5).

[23] W. H. Marshall and S. A. Talbot, "Recent evidence for neural mechanisms in vision leading to a general theory of sensory acuity," *Biological Symposia*, Edited by J. Cattell, Vol. VII, Visual Mechanisms (Cattell Press, Lancaster, 1942).

that a static condition could be maintained over an appreciable period of time, it is likely that the cone receiving the stimulus would become fatigued so rapidly that the response would decrease to a low or possibly a subliminal value. Moreover, if the optical characteristics of the eye were such that an image of a narrow straight, bright line could have a width of less than a cone diameter, it might be expected that, since the cones are not arranged in perfectly straight lines, the sensory interpretation would be a wavy line and the same interpretation might be expected in the case of a straight boundary between a light and a dark area. Actually, there is no evidence of irregularity of receptor arrangement in our usual perception of straight lines or straight boundaries.

It might be assumed that since we are forced to abandon a static conception of the retinal image as related to the receptors, it is of little importance to consider further the dimensional characteristics of these receptors. This, however, is not the case, since even with the dynamic interpretation the size of the receptors relative to the elements of the retinal image will determine the time *rate of change* of the illuminance incident on the receptors which, as much evidence indicates, is an important determinant of the magnitude of the neural response evoked by a stimulus.

Figure 3 may now be used as a basis for computing the variation in the illuminance incident upon the various retinal receptors, and by making certain assumptions concerning the nature of the physiological nystagmus, approximate values for the rate of change in illuminance can be obtained. The frequency and amplitude of the physiological nystagmus upon which our computations are based are in accordance with the findings of Adler and Fliegelman.[21] These investigators report that when an effort is made to fixate precisely on a fine vertical line there are three types of horizontal eye movements.* The first they designate as "rapid shifts," which occur at intervals of from 1 to 2 seconds and have amplitudes of approximately 30 minutes of arc corresponding to a movement of the image on the retina of approximately 100μ. The second, termed "waves," are "inconstant in direction, extent, and duration." The amplitude corresponds to movements on the retina of from 0.008 to 0.016 mm. This amplitude, therefore, varies between 2.5 and 5.0 minutes of arc. They then state "each wave lasts less than 0.2 second." Since they have expressed one characteristic of these waves in terms of amplitude, we assume that the 0.2 second corresponds to the transit time, that is one-half cycle rather than a complete cycle.

The third type of motion which they refer to as "fine vibratory movements" is described as "small trembling movements, averaging in number from 5 to 10 in 0.1 second." Again we assume that this refers to a single transit rather than to the complete cycle. These movements, therefore, occur at the rate of from 25 to 50 cycles per second. The authors suggest that these movements "probably represent the vibration frequency of the extra-ocular muscles." If this is true, the vibratory motion should be of the simple harmonic type in which the velocity at which an image of a point moves on the retina is 0 at the extremes of the cycle and maximum at a point midway between the extremes. The amplitude of this vibration has an average value on the retina corresponding to 2 minutes and 14 seconds of arc. They state further that because of the inability of their photographic recording mechanism to represent faithfully the extreme limits of each excursion of the mirror, the values of amplitudes given are probably underestimates rather than overestimates.

One or two other references to eye movements during attempted fixation have been found. Marx and Trendelenburg[24] report that the value for the amplitude of these motions varies from 4 to 5 minutes of arc. This is about the same as the value reported by Adler and Fliegelman for the low frequency vibration but twice as great as that for the high frequency. Considering the difficulty of making such measurements we consider it excellent agreement. These authors do not make any statement of the frequency.

* No data are available on the possible vertical or other directional movements, although these investigators state that their equipment is capable of measuring motion in other directions and that such findings will be reported at a later time. We have not been able to locate these reports in the literature.

[24] E. Marx and W. Trendelenburg, "Über die Genauigkeit der Einstellung des Auges beim Fixieren," Zeit. für Psychologie, Abt. II, Sinnes-Physiologie **45-46**, 87 (1911-12).

Yourievitch[25] states that the duration of these movements varies from 0.02 to 0.06 second. It is not clear from the text whether these values refer to the time for a complete cycle or for a single transit (half-cycle). In any case, the frequencies are of the same order as those reported by Adler and Fliegelman. This author makes no statement regarding amplitude. The two references taken together support fairly well the values for frequency and amplitude given by Adler and Fliegelman.

In Fig. 3 the points X and Y represent the extremes assumed for the high frequency movement of the central cone, a. This distance we have taken as 2.4 minutes, a value slightly greater than that given by Adler and Fliegelman, but probably justified according to their own statement because the peaks of their records were not well recorded. In computing, we have assumed a value of 0.02 second for a single transit from X to Y, corresponding to a vibration frequency of 25 cycles per second, and we have also assumed a simple harmonic motion. It must be remembered that these amplitudes and frequencies represent average values and that the horizontal movements of the eye during attempted fixation should not be considered as having a single definite frequency and amplitude. In fact, Marshall and Talbot[23] (p. 138) describe this physiological nystagmus as a "fluctuation with approximately a statistical occurrence of various amplitudes." However, the experimental evidence for such a statistical distribution of amplitudes is by no means conclusive. In fact, since Adler and Fliegelman measured only the horizontal component of the vibrations, a variable horizontal amplitude would be expected even if the actual amplitude is approximately constant, but variable in direction. It seems reasonable to assume that when an attempt is made to fixate a *point*, the vibratory motion of the eyeball will be distributed in all directions, giving rise to the horizontal amplitudes of variable magnitude distributed possibly according to a sinusoidal function rather than a purely random one.

In Fig. 3 the cone group represented by the row marked L shows the central cone, a, at the left-hand extremity of its movement and the cone grouping, R, shows the cone, a (and its adjacent cones), at the right-hand extremity of its motion. It is now possible to compute the average illuminance incident on each of these cones at various positions in their vibratory cycle. This has been done and the results are shown graphically in Fig. 4. The ordinates give the values of the relative illuminance, I_r, incident on the various cones, while the abscissas represent time (in milliseconds) reckoned from ten at the instant when they start moving to the right relative to the position of the geometrical boundary, MN. The curves show the relative illuminance incident on the cones, $-f$, through a to $+f$, as designated by these letters, which correspond to the labeling in Fig. 3. As the central cone, a, moves to the right from its extreme left position (Fig. 3), the illumination increases slowly at first, then more rapidly, and again slowly, reaching a maximum at the end of 20 milliseconds, which is the time we have assumed to be occupied by a one-half cycle. During the second half of the cycle the cones move to the left, the illuminance decreasing according to the curves in Fig. 4. The curves in Fig. 4 serve to give direct evidence for the amplitude of illuminance changes on the various receptors located near the geometrical boundary between the light- and the dark-field areas. The gradients of these curves indicate the rate at which this change in illuminance occurs.

In Fig. 5 the gradients have been plotted as a function of time, the ordinates in this case being the values of dI/dt. It will be seen that for Cone a the maximum rate of change occurs when this cone crosses the geometrical image of the boundary, MN. At this point the illuminance is changing at the rate of approximately 21 percent (of B_1-B_2) per millisecond. The adjacent cone, b, has a maximum rate of change slightly less, but it occurs not at the instant when it crosses the geometrical image of the boundary but at a point 1.5μ to the right. Cone c shows a still lower maximum rate of change, and Cone d an even lower. Moreover, the total amplitude of luminance variation for d is appreciably depressed, Fig. 4; and for cones lying to the right of d decreases rapidly until the sixth cone, f, is subjected to an illuminance change of only about 15 percent of B_1-B_2.

[25] S. Yourievitch, "Les Principaux Caractères de Mouvements Oculaires," Comptes rendus **188**, 937 (1929).

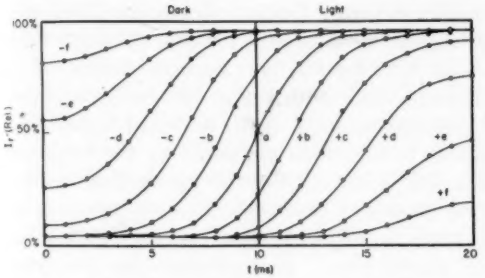


FIG. 4. Curves showing the change in illuminance incident on various retinal cones as they move, by virtue of the physiological nystagmus, relative to the image of the boundary between a light and a dark area.

If this picture represents more or less faithfully the condition when the boundary between a dark and a light area is fixated, we have a condition where several "rows" of cones parallel to the geometrical image of the boundary are subjected to a rapidly changing illuminance, the frequency of which is the same for all rows of cones. However, the rate of change of illuminance is maximum for the central row and decreases slowly and then rapidly for the more remote rows. Likewise, the amplitude of the illuminance change is maximum for the central row, and this also decreases slowly at first and then rapidly for the more remote rows.

There is much evidence in the literature to indicate that the magnitude of the neural response which occurs when a receptor is stimulated is conditioned by the suddenness with which the stimulation is applied, or by the rate of change in the intensity of the stimulus during application, and, further, that if the stimulus is of appreciable duration, there is a relatively rapid fatigue effect which decreases the magnitude of the neural response even though the stimulus remains constant. This has been shown strikingly by Hartline and Graham.[26] The mechanism suggested by Fig. 3 is a relatively efficient one for applying a series of rapidly changing stimuli to many receptors, and hence the system should also be highly efficient for the generation of neural responses.

A markedly different picture is obtained when a similar analysis is applied to a retinal area in which the center distance between adjacent cones is appreciably greater than the 1.5-μ value used in the construction of Figs. 4 and 5. To illustrate this, Fig. 6 is given. The curves in Fig. 6 show the amplitude of illumination on the cones in the immediate vicinity of the geometrical image of the boundary between a light and a dark area, a value of 4.5μ being assumed for *CD*. Reference to Fig. 2 indicates that according to Polyak's measurements the center distance becomes 4.5μ at just a little beyond one-degree departure from the center of the fovea. This is approximately at the limit of the rod-free area. Curve *a* in Fig. 6 shows the illuminance amplitudes to which the central cone, *a*, is subjected. Curves *b* and *c* give the same information for the first and second cones to left and right. It will be seen that the illuminance amplitude has decreased to a relatively very small value for the *c* cones. The illuminance gradients for this same condition are shown in Fig. 7. It will be noted that the central cone, *a*, has a maximum gradient of approximately 15 percent per millisecond compared with that of 21 percent for the 1.5-μ cone (see Fig. 5). The decrement in gradient is rapid for the *b* and *c* rows. In fact, the gradient to which the *c* cones are subjected is so small that it is doubtful that a supraliminal neural response will be produced.

Since we know that the value of *CD* is a minimum at the exact center of the fovea, this part of the foveal mosaic is of highest efficiency for producing high rates of change in the illuminance incident upon the individual receptors and, as experience shows, the acuity, resolving power, and brightness discrimination are maximum when attention is fixed on the test object, thus bringing the image of those visual field elements onto that part of the fovea where the receptors have the minimum size and separation.

It is evident from a consideration of Figs. 3–5 that the retinal receptors lying at a distance of more than approximately four or five cone diameters from the geometrical image of the boundary, *MN*, are subjected to very little, if any, change in illuminance when attention is fixed on the boundary line. It can be assumed, therefore, that the neural responses from relatively few receptors are responsible for the perception of the boundary. The actual number involved in the perception of

[26] H. K. Hartline and C. H. Graham, "Nerve impulses from single receptors in the eye," J. Cell. and Comp. Physiol. 1, 277 (1932).

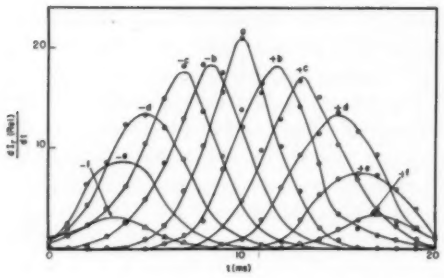


FIG. 5. The illuminance gradients to which various retinal cones are subjected as they move, by virtue of the physiological nystagmus, relative to the image of the boundary between a light and a dark area.

this boundary is obtained, approximately, by multiplying the length of the boundary line, in terms of foveal cones, by 11 or possibly 9, since we have no definite evidence to indicate whether the illuminance change (including both amplitude and rate) to which the cones, $-f$ and f, are subjected is of sufficient magnitude to produce a supraliminal neural response.

Since it is almost certain that the cone size (or spacing) is an important determinant of visual acuity, resolving power, etc., and since the size of receptor varies appreciably within the central fovea, it is desirable to determine with some degree of certainty just exactly what part of the central fovea is active in the determination of visual acuity and in the judgment of disappearance of detail within the visual field, this latter judgment being one which must be made in the measurement of graininess.

We shall, therefore, revert to a further discussion of receptor sizes and spacing, especially within the retinal area which probably is of greatest importance in the determination of the graininess of photographic deposits by the *blending* method. In Fig. 8 is a schematic, idealized diagram showing the relationships between the various retinal foveal regions, as defined by Polyak. The circle having the radius A represents the "small central island" defined by the line A, Table II. The circle having the radius B corresponds to the region defined by the line B in Table II. The region characterized by the data in the line C of Table II, the *rod-free* area, is represented by the circle of radius C.

The visual field used in our measurements of graininess varied[1] from 30 minutes to 2.6 degrees in angular subtense, the average diameter being about 1.5 degrees, an average radius of 45 minutes, Circle D in Fig. 8. Hence, if the retinal regions represented by the circles of radius A, B, and C, in Fig. 8, be projected by the refracting elements of the eye onto the graininess fields at their blending distances, the dimensional relationships will be as shown in the figure where the dotted circle, E, represents the graininess field of least angular subtense (30 minutes). In this case the judgment of detail disappearance must depend almost entirely upon the retinal receptors within the small central island where the average CD is 1.9μ. There are 2500 retinal receptors within this area. For this case, then, it can be concluded that the small central island is largely, if not entirely, responsible for our appraisal of the inhomogeneities, or the lack thereof, in the field of smallest angular dimension which was used in our experimental work.

In making a judgment of detail disappearance (blending distance) when the field has an angular subtense of 1.5 degrees, the observer consciously scans the entire field. That is, he "looks at" all parts of the field, in succession. Thus, the visual axis represented by O in Fig. 8 must move in a more or less random fashion so that almost every part of the field is brought to, or at least very near to, the visual axis. The diameter of the A region is one sixth that of the 1.5-degree field and to us it seems reasonable to conclude that even in this case the final judgment of detail disappearance is probably conditioned largely by the structure and functioning of the A region or a part thereof.

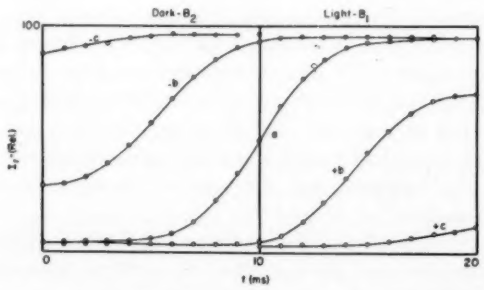


FIG. 6. Change in illuminance on retinal cones assuming a cone diameter of 4.5μ instead of 1.5μ which was used in the computation of the curves in Fig. 4.

Polyak (reference 8, Fig. 99) shows the relation between visual acuity and the angular separation between the acuity test object and the visual axis (fixation point). Here again the abscissa scale is such that the exact shape of this function cannot be determined precisely for small angular values. The curve does indicate, however, that acuity drops from about 18 seconds on the axis to about 60 seconds at a point 1 degree off axis, or from 100 percent to 30 percent.

We have been unable to find in the literature very many data concerning the variation of visual acuity with angular departure from the fixation point within ±1 degree and it is this region in which we are here chiefly concerned. The subject has been studied by Wertheim[27] but apparently he made no determinations at small angular departures from the fixation point. Weymouth and his co-workers[28] published some values covering the region from 0 out to an angular departure of about 1.5 degrees. The test object which they used, however, had an angular subtense of a little more than 10 minutes of arc. There is some doubt, therefore, as to whether their values of acuity represent the average for the whole test object or for a part of it lying nearest to the visual axis. Moreover, the test object used does not appear to us to be very satisfactory for this particular problem. Wertheim and his co-workers are quite aware of these shortcomings and state that the slope of their

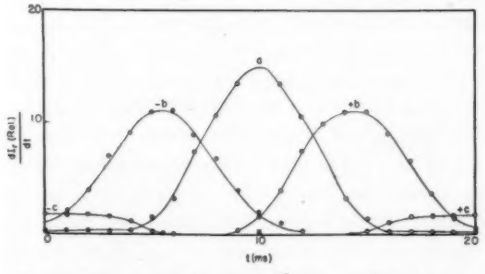


FIG. 7. Curves showing rate of change in illuminance on retinal cones having a diameter of 4.5μ instead of 1.5μ which was used in computing the curves shown in Fig. 5.

[27] T. Wertheim, "Ueber die Zahl der Scheinheiten im mittleren Theile der Netzhaut," Arch. f. Ophthalm. (Graefe) **33** (2), 137 (1887).

[28] F. W. Weymouth, D. C. Hines, L. H. Acres, J. E. Raaf, and M. C. Wheeler, "Visual acuity within the area centralis and its relation to eye movements and fixation," Am. J. Ophthalm. **11**, 947 (1928).

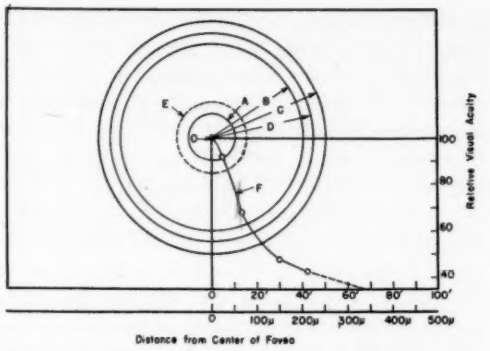


FIG. 8. Relation between acuity and angular separation from the exact center of the fovea shown with reference to the three foveal regions as defined by Polyak.

acuity curve may not represent the facts precisely. Their conclusions can be best stated in their own words:

> In the light adapted eye a uniform sensory gradient is shown to exist in this central area similar to that found in the entire retina (cf. Wertheim, Aubert, Fick, and others). The visual acuity attains a sharp maximum at the axis of fixation; it decreases rapidly but regularly in all directions; it shows no breaks or marked variations in rate of change at the margins of any of the known anatomical areas (fovea, rod-free area, pigmented area of macula, or nonvascular area).

We have made some measurements of acuity to establish this relationship with greater certainty in the region between 0 and a 45-minute angular departure from the fixation point. A standard Landolt ring cut out of opaque black paper was used as the test object. It was placed at the center of a large transluminated diffusing screen (angular subtense of 10 degrees) which, from the observer's point of view, had a luminance of 26 foot-lamberts. A set of readings was made by each of two experienced observers with the test object located at various angular distances from the fixation point. The results are shown graphically by Curve *F* in Fig. 8. The ordinate scale, relative visual acuity, is given at the right. Extrapolated to 1 degree, the results agree fairly well with that read from Fig. 99 of Polyak.[8] At a point in the visual field 10 minutes off axis, the visual acuity was found to be 75 percent of that at the fixation point, and at 5 minutes off axis, the value is 92 percent. Now, our experience indicates that the viewing distance at which

graininess disappears from the visual field can be determined to within about ±5 percent. A loss of 5 percent in acuity is encountered at a point 3.5 minutes off axis. In our opinion this may be interpreted to indicate that the neural responses from the foveal receptors lying within a circle, having a radius of 3.5 minutes (17.5μ) centered about the fixation point, control the judgment of disappearance (or appearance) of graininess. The *CD* value, read from the curve in Fig. 2, at 3.5 minutes off axis is 1.35μ, and the average within the circle having this radius probably is somewhat less. However, by using *CD* equal to 1.35, we find that this area is inhabited by about 600 cones, the diameter of this circle corresponding to a row of 26 cones in line.

Weymouth and his co-workers have advanced an interesting idea concerning the relationship between acuity gradient and the act (of fixation.) They recognize that this idea is not original with them since they state:

> The idea that variation of acuity, or sensory gradient, is the basis of fixation is not a new one. Javal[29] propounded it as long ago as 1896, and Duane[30] (pages 267, 290, 342) reiterated it in 1924, although realizing the lack of information regarding sensory gradient within the fovea. Marx and Trendelenburg[31] believe their "fixation area" is based on differences of acuity within the area centralis, the image being brought back to the area of maximum acuity as soon as it becomes of appreciably less than the maximum distinctness.

In their discussion of experimental work, Weymouth and his co-workers present a convincing argument for this theory and it seems quite likely to us that the sharp acuity gradient which we and others have found to exist is, in fact, the most probable controlling factor in fixation. It is interesting to note that Marx and Trendelenburg in their investigation of the accuracy of the act of fixation have reported values for the amplitudes of the physiological nystagmus in good agreement with those found by Adler and Fliegelman. They too observed relatively high frequency oscillating fluctuations of about 2 minutes in amplitude on which are superimposed irregular fluctuations of 4 to 5 minutes. They interpret these larger movements as of a corrective nature arising when the image departs by more than 2.5 minutes from the center of a fixation area. They conclude that the accuracy of fixation as a whole is approximately ±2.5 minutes. All of these data have a direct bearing on the mode of functioning of the visual mechanism when a judgment of blending distance is made in connection with graininess measurements.

Another type of acuity experiment has been made by us which throws some additional light on the number of foveal receptors concerned in detecting just perceptible visual field elements of varying shape. For this purpose a series of special test objects was prepared. These are rectangular in shape, the ratio of length, l, to width, w, varying from 1.08 to 4050. The values of l, w, l/w, and a (area) are shown in the columns thus designated in Table III. These test objects were made by photographing, at a large reduction ratio on high resolution (resolving power equal to 1000 lines per mm) and high contrast photographic plates, large-scale drawings of the test-object shapes desired. The test objects thus obtained were opaque black silver deposits on a support of high transmittance, the transmittance ratio being greater than 1000. These were mounted, one at a time, in the center of a large transluminated diffusing screen (visual angular subtense equal to 10 degrees) which, from the observer's viewing station, had a luminance of 26 foot-lamberts. The distance between the test object and the observer was then adjusted to the threshold of visibility. Three experienced observers, all of whom have approximately normal vision, each made several settings on each test object. The data were averaged and the angular subtense, α, of l, and the angular subtense, β, of w, were computed. These are shown in columns α and β in Table III. The results are shown graphically in Fig. 9. To aid in visualizing the relation of the geometrical images of these test objects, at the threshold of visibility, to the retinal mosaic, the data have been plotted as a two-limbed curve, A and B. The horizontal distance between the two limbs gives the angular subtense, α, of l (in minutes) on the abscissa scale. The ordinate values give the angular subtense, β, of w (in seconds). The circle, C, represents the boundary of the small central island (Line A, Table II), its dimension being

[29] E. Javal, *Manuel du Strabisme* (Paris, Masson, 1896).
[30] R. Duane, Fuschs' *Textbook of Ophthalmology*, eighth edition (Lippincott, Philadelphia, 1924).

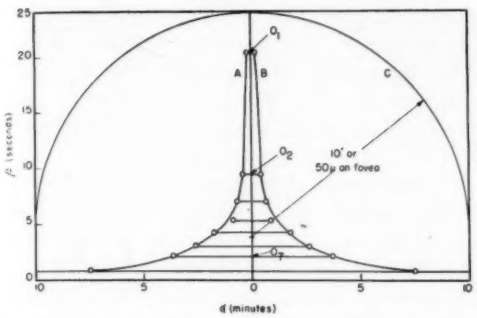


FIG. 9. Illustrating the sizes of the geometrical images on the retina for test objects at the threshold of visibility.

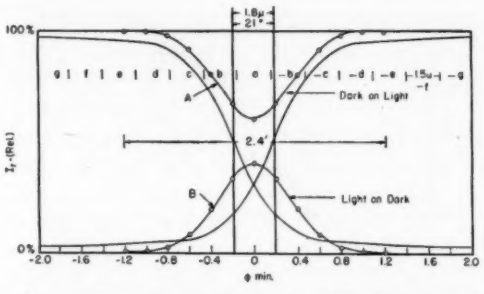


FIG. 10. Curve *A*, the distribution of illuminance on the retina for a dark test object, subtending an angle of 21 seconds, on a light background of large angular extent. Curve *B*, the distribution of illuminance on the retina for a light test object, subtending an angle of 21 seconds on the retina, on a dark background of large angular extent.

read from the abscissa scale. As drawn, the center of the fovea corresponding to the fixation point in the visual field is at 0. In judging the threshold of visibility, the fixation point must be considered as moving onto the various test objects at O_1, O_2, $\cdots O_7$, etc., in each case Circle *C* moving so that it is centered about the appropriate fixation point.

In the last two columns of Table III are shown the dimensions of the geometrical image on the retina, at the threshold of visibility, in terms of cone separation, *CD*. In making these computations a value of $CD = 1.35\mu$ was used for all the test objects.*

It will be seen that for *TO* (test object) No. 1, which is approximately square ($l = w$), the geometrical image on the retina does not entirely cover a group of four cones. However, because of diffraction and an assumed amplitude (in all directions) of ± 1.2 minutes for the high frequency (25 cycles per second) physiological nystagmus, the diameter of the roughly circular area on the retina, the nystagmic image, within which the receptors are subjected to an appreciably decreased (and rapidly changing) illuminance as compared to the surrounding visual field, is approximately 18 *CD* (roughly 5 minutes of arc or 25μ on the retina).

TABLE III. The visibility of rectangular test objects varying with respect to the length-width ratio.

No.	Test object—*TO* l (mm)	w(mm)	l/w	a_{mm^2}	Geometrical image Angular α'	β''	*CD* l	w
1	1.07	0.98	1	1.05	0.36	20.4	1.44	1.36
2	2.48	0.49	5	1.21	0.81	9.36	3.24	0.62
3	3.54	0.34	10	1.20	1.33	7.01	5.30	0.47
4	5.03	0.25	20	1.26	1.75	5.28	7.00	0.35
5	7.51	0.15	50	1.13	3.57	4.25	14.3	0.28
6	10.0	0.10	100	1.00	5.21	3.05	20.8	0.20
7	17.6	0.081	218	1.43	7.45	2.06	29.8	0.14
8	40.0	0.037	1082	1.48	14.9	0.83	58.0	0.055
9	59.9	0.023	2610	1.38	31.9	0.74	128	0.049
10	150	0.044	3410	6.60	39.3	0.70	160	0.047
11	150	0.037	4050	5.55	45.7	0.68	183	0.045

* We consider 1.35μ to be the most probable average of *CD* for the central area of the fovea having an angular subtense of 15 minutes (7.5 minutes on each side of the fixation point).

The distribution of illuminance along a diameter of the diffraction image when *TO* No. 1 is at "threshold visibility" is shown in Fig. 10, Curve *A*. As pointed out by Byram[10] and others, the distribution of illuminance on the retina in the diffraction image of a straight dark line on a light background can be obtained by combining the distributions in the images of two boundaries, one between a light to a dark and the other between a dark to a light area, the light areas being separated by the width of the dark line. Curve *A*, in Fig. 10, was obtained in this way. Since *TO* No. 1 is approximately a square dark area on a light background, it may be considered to consist of a very short part of a dark line. For the purpose of this discussion the assumption is made that this distribution represents the distribution along any diameter of the roughly circular diffraction image on the retina. It should be noted that when the square dark spot on a light background (*TO* No. 1) is at the threshold of visibility it is not perceived as a square but has the same appearance as a circular dark object on a light background.

In Fig. 11 are shown the amplitudes of illuminance to which the cones, *a* to *g* (or *a* to $-g$), are

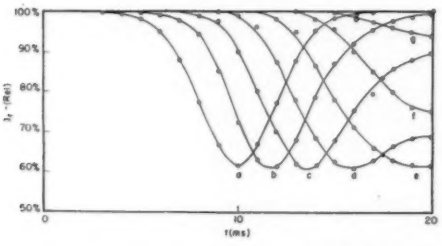


FIG. 11. Curves showing the amplitudes of illuminance to which retinal cones are subjected during one-half cycle of the physiological nystagmus.

subjected during one-half cycle of the physiological nystagmus. The cones, *a* to *d*, inclusive, each cross the geometrical images of two boundaries between light and dark areas and hence in each half-cycle are subjected to both negative and positive illuminance gradients. Cone *g* is subjected to a relatively small illuminance amplitude. Unlike the condition illustrated in Fig. 4, there are four (instead of two) cones on each side of the central cone, *a*, for which the illuminance amplitude is approximately the same as that for Cone *a*.

Curves *a* to *g*, inclusive, in Fig. 12 give the illuminance gradient, dI (relative)$/dt$, for the cones corresponding to the lettering of the curves. These curves cover one-half cycle of the nystagmic movement. It will be seen from Figs. 11 and 12 that there are five (possibly six) cones in every direction from *a* which are subjected to appreciable illuminance amplitudes and temporal gradients. Thus, while the *TO* No. 1 is fixated, all the cones lying within a roughly circular area on the retina, centering about the fixation point, are subjected to rapid changes in illuminance. The receptors lying outside of this area experience no change in illuminance. This area has a diameter of 11 (or possibly 13) *CD* and is inhabited by approximately 110 cones. Thus, the area of the retina which is responsible for the perception of *TO* No. 1 is relatively very small, having an angular subtense of only 3 minutes and 18 seconds.

The conditions illustrated in Fig. 10 are typical of those which exist when an observer views a developed silver image of relatively low density. This consists of rather widely separated opaque silver grains on a bright background. As the density increases, the number of grains per unit area increases (the distance between adjacent grains decreases) and the densely clustered grain clumps at the same time become greater in size. When a high density is reached, we encounter a reversal of background and figure, the ground now being dark with relatively small clear spaces, "holes," rather widely spaced, and having the same luminance as the background in the case of a low density. For these "holes" the distribution of illuminance on the retina is also obtainable by the construction shown in Fig. 10, Curve *B*, where the hole is assumed to have the same dimensions as *TO* No. 1.

It is well known that a light figure on a dark ground gives greater visibility than a dark figure of the same angular dimensions on a light ground, assuming the same ratio of figure to ground illuminance. Thus, under conditions similar to those used in obtaining data in Table III, a small light spot on a dark background has at the threshold an angular subtense of only 11 seconds compared with 22 seconds for the reverse case. This is attributed by some authors to "irradiation." This term does not seem to be precisely defined and apparently is not used in the same sense by all investigators in this field. However, in some cases irradiation implies a spreading of neural excitation within the retina itself. This spreading is assumed to occur through the lateral synaptic connections between the bipolar and ganglion cells prior to the passage of the nerve impulses into one of the fibers of the optic nerve.

Although this type of "irradiation" may and quite probably does occur, at least in certain parts of the retina, the experimental evidence is not at all conclusive with respect to the existence of such interaction between adjacent receptors in the immediate center of the fovea. Moreover, to us it seems unnecessary to assume such spreading in order to explain the observed facts. Thus, in the case of *TO* No. 1, all the cones in the foveal region surrounding the nystagmic image on the retina are exposed continuously to a constant relatively high level of illuminance and hence are in a partially fatigued or somewhat depressed state of sensitivity. When, because of the physiological nystagmus, they are brought into the region of lower illuminance, the rate of change of

illuminance produces less neural response than if they had been preadapted to a lower level of illuminance. In the case of a light test object on a dark ground, the foveal cones lying outside of the diffraction image on the retina are subjected to a relatively low level of illuminance and hence are in a state of relatively high sensitivity. Now when, by virtue of nystagmus, they move into the region of higher illuminance, the same actual rate of change of illuminance produces a greater neural response, and hence a test object having a smaller angular subtense becomes perceptible.

The results obtained with other test objects used in obtaining the data in Table III have somewhat less direct significance for the graininess problem. This follows because the visual field elements in a developed silver image are not of the "fine-line" or "long-rectangle" types. They consist almost entirely of roughly circular, approximately equilateral, triangular, regular hexagonal, elliptical (with the ratio of major to minor axis relatively small), and irregular but closely clustered groups of these forms. The field elements seen in a developed silver image are seldom of the long slim or thin type.

However, the data relative to the test objects used in obtaining Fig. 9 are of some interest here, since they throw considerable light on the mode of functioning of the visual mechanism. It will be seen (Fig. 9) that as the angular subtense, β, of w decreases, the angular subtense, α, of l increases. The test objects change rapidly from the square shape to long, thin rectangles and finally to what are better described as "fine lines." In the case of the fourth test object, the width of the geometrical image on the retina has decreased to 5.28 seconds, corresponding to about one-fourth of *CD*, while the length is 1.75 minutes, which is equivalent to approximately 6.5 *CD*. Again, because of diffraction and physiological nystagmus of ± 1.2 minutes in all directions, the dimensions of the area on the retina within which the receptors are subjected to a changing illuminance becomes about 17 *CD* wide by 22.5 *CD* long. This area is inhabited by about 440 cones. Hence, more cones come into action in order to permit the narrow long rectangle to be seen as compared with the square test object. In the case of Test Object No. 7, which is now of the "fine-wire" type, the area covered by the diffraction and the nystagmic image is about 17 *CD* wide by 75 *CD* long, involving approximately 1400 cones. For *TO* No. 10, the retinal area involved increases to about 17 *CD* by 200 *CD*, making a total of approximately 3400 cones which must be involved in the just-perceptibility of this fine line. Thus, in order for a very narrow line to be seen, it must be long enough to bring into action more and more retinal receptors.*

Voluntary Eye Movements

In making a measurement of graininess in terms of blending distance, the observer voluntarily directs attention to various parts of the

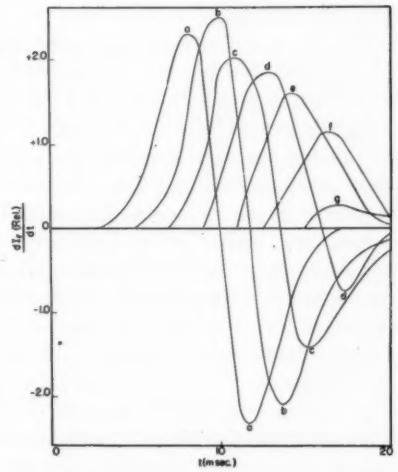


FIG. 12. Curves showing rate of change of illuminance incident on retinal cones during one-half cycle of the physiological nystagmus.

* In making these computations we have assumed that the observer attempts to fixate the test object at the center of its long dimension and that the physiological nystagmus is equally distributed in all directions about this point. This assumption is probably unwarranted since it is likely that at the threshold the fixation point may drift appreciably in the direction of the line length. Observers making visual-acuity measurements generally state that at the threshold the visibility of the test object "comes and goes." This appearance and disappearance of the test object is sometimes attributed to changes in "attention." Is it not possible that this phenomenon is caused by the maximum visual acuity at the immediate center of the fovea and a rapid falling off in all directions from that point? Hence, the test object is seen when the image of the test object is formed at the exact center of the fovea but becomes invisible just as soon as there is motion of the eyeball sufficient to move that image even a very small distance in any direction away from the exact center.

graininess field. It is of interest to consider the rate at which the scanning takes place. Concerning such eye movements, Duke-Elder[31] states: "On the average in small movements the speed may be taken as covering an angle of 100 degrees to 200 degrees per second." If the excursion between fixation points is relatively long this velocity may rise from 200 to 500 degrees per second. The scanning speed is greater when excursions of large magnitude are contemplated and the velocity in general is higher in the middle of the pathway, diminishing as the contemplated fixation point is approached.** It is well known that in many ordinary visual tasks, such, for instance, as reading a line of print, the movements of the eyes are not smooth but consist of a series of jumps interspersed with pauses of short duration.

In examining a visual field having an angular subtense of only 2 or 3 degrees, it is probable that the lowest velocity mentioned by Duke-Elder should be assumed. With a velocity of 100 degrees per second and a cone, CD, of 1.5μ, the image on the retina of a point in the visual field traverses 20 cones per millisecond. Thus, the geometrical image of a boundary, MN (Fig. 3), would traverse (if perpendicular to the direction of the eye movement) a 1.5-μ cone in 0.00005 second, or, if we consider the diffraction image of that boundary to extend from 4 percent to 96 percent of B_1-B_2, then that image traverses each cone in 0.00045 second. With the luminance differences ordinarily encountered in a graininess field at the blending distance, the fluctuating stimulus to which the retinal receptors are subjected under these conditions is almost certainly subliminal. Thus, during the transfer of attention from one part of the field to another, it is doubtful if any detail is perceived.

The judgment of blending distance usually is based on a series of visual operations similar to those represented schematically in Fig. 13. The boundary circle represents a hypothetical field having an angular subtense of 2 or 3 degrees. It is filled with a magnified image of a developed silver deposit, the magnification being such that

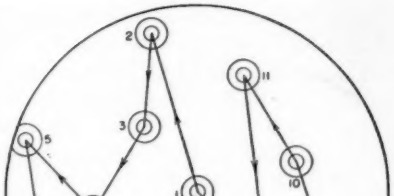


FIG. 13. Schematic diagram showing the way in which a visual field is scanned in making a judgment of graininess threshold.

the graininess is at the just detectable point. The observer in first regarding this field probably looks at a point lying somewhere near its center, (1). Attention may rest there briefly, during which time the physiological nystagmus of high and intermediate frequency causes the central point of the central fovea to move over the small fields in various directions. The smaller of the two small concentric circles represents the amplitude (angular subtense = 2.4 minutes) of the higher frequency (25 cycles per second) physiological nystagmus, while the larger of the two concentric circles represents the nystagmic movement of greater amplitude (angular subtense 4 to 6 minutes) and lower frequency. The rate of change of luminance incident upon a visual receptor is approximately the same for the two amplitudes since the greater amplitude has a lower frequency which compensates fairly closely for the higher frequency in the case of the movements of smaller amplitude.

After the observer has decided whether or not graininess is visible, his attention is then shifted, as, for instance, to the point (2) where, again, there is a pause. This operation continues until the observer has examined several areas in the field, as indicated by 1, 2, 3, etc., and possibly his attention may return one or even more times to certain areas of particular interest. The time during which attention is fixed at a particular spot may be termed a "fixation pause." The duration of these pauses may vary considerably,

[31] W. C. Duke-Elder, *Textbook of Ophthalmology* (Morsley Company, St. Louis, 1931), Vol. I.

** An excellent bibliography covering the literature of this subject will be found in reference 31, pages 611, 612.

but from a careful consideration of this operation it is estimated that they probably vary from a few tenths of a second upward to possibly 2 or 3 seconds. The judgment as to the appearance or non-appearance of graininess we believe is based entirely upon the responses of the retinal receptors during the fixation pauses. The involuntary action of the eye during the fixation pauses may be likened to the effort to feel texture on a non-smooth surface. The tip of the finger, if laid on such a surface and held in a perfectly static position, gives a very inferior report concerning the texture of the surface; but as soon as it is moved back and forth in various directions over the surface, the texture is perceived.

It seems highly probable that a relatively small group of cones, not more than 60μ† (possibly less) in diameter, clustered around the center of the fovea (fixation point), are largely responsible for the final judgment as to the visibility or invisibility of graininess. At the edge of a field of this size, having an angular subtense of 12 minutes, the visual acuity has decreased to a value which is approximately 90 percent of that on the fixation axis. In this region of the retina, the cones have an average *CD* of about 1.5μ (or less) and the image on the retina when graininess is just visible, as it is swept across the retinal mosaic by the nystagmic motion, must produce rates of change of illuminance on the individual receptors sufficient to evoke just supraliminal neural responses.

If this assumption is correct, then it appears reasonable to conclude that if a granularity measurement is to have a significant correlation with the judgment of graininess, the size of the scanning spot used in the granularity measurement must be closely related to the size of the retinal receptors which determine the judgment of blending distance. In our opinion this size of the retinal receptors is approximately 1.5μ, although it is impossible to predict with certainty exactly what the diameter of the scanning spot should be. However, it is possible to draw some conclusions from available evidence which we shall introduce at a later point in this discussion.

† Assuming hexagonal packing and $CD = 1.5\mu$, this area is inhabited by approximately 1450 cones.

Some Aspects of the Neural Response

Since we can make no pretense of being intimately familiar with this field, it is with some trepidation that we attempt even an elementary discussion of this subject. Nevertheless we believe that an understanding of at least some of the characteristics of the neural response may assist materially in choosing a technique for the measurement of granularity which will yield results from which graininess can be predicted with certainty. Therefore, in the following paragraphs we shall attempt to discuss what is known concerning the minute structure of the retina, the synaptic connections between the retinal neurons, and the characteristics of the nerve pulses which are propagated from the photoreceptors along the afferent visual pathway into the cerebral cortex. This discussion will be limited to those aspects of the anatomical structures and nerve responses which we believe are most relevant to the perception of graininess.

There is quite general agreement among authorities in this field that the neural responses following the stimulation of the retinal receptors, the rods and cones, by radiant energy arise as the result of photochemical reaction which takes place when the radiant energy is absorbed by or in the immediate vicinity of the receptors. For our problem we are not particularly concerned with the exact nature of this photochemical process. We are, however, interested in the manner in which the nerve pulses are conducted from the point of origin along the visual pathway into the cerebral cortex.

We shall look first at the information which can be obtained from the results of histological and anatomical studies of the structure of the retina and its associated nerve systems. For information on this subject we shall rely largely on the work of Polyak.[8] The cones (or rods) are the first link in the chain of neurons which forms the afferent visual pathway. We are concerned only with the cones since they almost certainly are the receptors which are responsible for the perception of graininess. This follows because the judgment concerning the presence or absence of graininess in the visual field is made by looking directly at the test field. Hence, the image of the test field is formed at and around the center

of the central fovea which lies in a rod-free region having an angular radius of approximately 50 minutes of arc. At the edge of this field, visual acuity is only 40 percent of that on the axis, and at a distance of 10 minutes of arc from the axis the acuity is down to 75 percent of the axial value. It is extremely improbable, therefore, that the rods in the retina can have any role whatsoever in the perception of graininess at the threshold level. Moreover, the judgment involved in setting the graininess instrument to the threshold of detail disappearance probably depends very largely, if not entirely, upon the neural responses of a relatively very small group of cones clustered immediately around the fixation point.

The cones are synaptically connected to the neurons which form the second link, namely, the bipolar cells which are a part of the retinal structure. In the words of Polyak (reference 8, page 178), "The cone bipolars possess outer expansions or dendrites which are placed horizontally and touch the cone pedicles from underneath. Their inner expansions, or axis cylinders, are in contact with the dendrites of the ganglion cells." The ganglion cells, also a part of the retinal structure, form the third link in the chain of neurons, the last link situated in the retina. The axis cylinders of the ganglion cells become the fibers of the optic nerve. These fibers are long and, after being carried by the optic nerve into the optic chiasma, continue without interruption along the optic tracts, finally terminating in the subcortical visual centers. Of these, the principal one is the lateral geniculate body. Here these fibers synapse with others which terminate in the occipital lobes of the cerebral cortex, the "striate area."

There are, however, other neurons in the retina. Among these of possible importance from our point of view are the *horizontal* or outer associational cells which probably are not involved in the transmission of neural impulses from the receptors to the cortex. For our purpose it is desirable to know whether the nerve impulses reaching the cortex from a single fiber of the optic nerve are caused by the stimulation of a single cone, or whether the responses of several cones are carried by a single fiber. There is little doubt that in some parts of the retina the synaptic connections between cones, bipolars, ganglion, and association cells are multiple, and hence some optic nerve fibers carry nerve impulses which result from the stimulation of more than one, and in some cases many photoreceptors. But, as we have said before, the perception of graininess seems to depend largely on the neural responses of a relatively small group of cones situated immediately around the exact center of the central fovea, the fixation point. While a comprehensive understanding of all visual phenomena requires a knowledge of how the entire retina and all of its parts function, our present concern is chiefly centered on the central portion of the fovea centralis. Concerning this point, Polyak has much to say. Thus, on page 179 we find:

> In the central area the ganglion cells are thought to be related to the bipolars singly, a circumstance which, together with a like individual relationship between the cones and the bipolars in the foveal region already mentioned, would account for the increase of the number and reduction of the size of the individually stimulable points—that is, greater acuity—in the parts of the fields of view near the fixation point.

And further, on page 248,

> In the central portion of the central area (*II*), including the fovea (*I*), an outer scleral expansion or "bouquet" of each individual or midget bipolar (*h*) is attached from below to each cone pedicle. In the regions mentioned, accordingly, with few exceptions, the *b-h* relationship is strictly monosynaptic or individual ("private").

In discussing in greater detail the synaptic relation between the midget ganglion and the midget bipolar cells, Polyak has this to say (page 326):

> Evidently, the extraordinary arrangement—the coupling of two conducting neurons by a single synapse, most frequent precisely in the axial region of the retina, where from physiological considerations it would have to be expected—serves a definite functional purpose. Apparently it acts as an anatomical safeguard which prevents an individual nervous impulse, originating in a particular cone and passing through a particular midget bipolar to its related midget ganglion, from spreading to adjoining midget ganglion cells. It is this intimate anatomical relationship between a single midget bipolar and a single midget ganglion (*h-s* synapse), on the one hand, and between a single midget bipolar and a single cone (*h-b* synapse), on the other hand—besides the presence of Müller's "radial fibers"—which, better than any other structural detail, explains the belief of the early investi-

gators in a direct continuity of the optic nerve fibers with the outward elements of the retina and in the termination of these fibers as rods and cones, a belief based upon teased preparations. After the introduction of modern histological methods this "continuity" was lost sight of, since most of these methods do not demonstrate well the existing synapses (exceptions are Golgi's and Ehrlich's methods). Through the identification of the midget ganglions (*s*) and of the midget bipolars (*h*) the problem of long standing, *viz.*, whether or not there are in the retina, and in the nervous system in general, arrangements which could be called "private" or individual synapses, or, in other words, whether there are the so-called "point-to-point" or "one-to-one" relationships of neurons, appears to be solved in the positive sense.

In Fig. 14 is reproduced one of the illustrations given by Polyak (Fig. 97) to illustrate certain types of synaptic couplings found at various places in the retina. Since we are primarily interested only in the rod-free area, the parts of Fig. 14 designated as *A*, *B*, *C*, and *H* represent conditions not applicable to our problem. And, in fact, the arrangement at *E*, but with the ganglion cell *m* omitted, is that which we conclude is found in the central part of the fovea. The three elements in this chain of neurons are the cone, *b*, the midget bipolar, *h*, and the midget ganglion cell, *s*. The numbers and letters in the horizontal column at the left of the figure are those used by Polyak for the designation of the layers of the retinal structure.*

According to Polyak, therefore, there is in the retina, at least one region where there exists an arrangement which he refers to as the pure or private cone system. While we do not find any definite statement by Polyak as to what he considers to be the angular size of this region, we think it is reasonable to conclude that it does include all of the relatively small region around the fixation point which is involved in the judgment of a blending distance. Concerning the nature of this pure cone system, Polyak speaks further, as follows (pages 389–391):

> The set of the retinal neurons which may be called the pure cone system is composed of the following three links (Fig. 97, *E, but with the diffuse ganglions, indicated by the m-ganglion, thought of as being absent*): the cones (*b*); the midget bipolars (*h*); and the midget ganglion (*s*), which in this conjunction may be considered a *cone ganglion*. In the central area almost entirely, and in the extra-areal regions progressively less so, the private cone system is organized strictly "spatially." Here each photoreceptor, a cone, is synaptically related to a single midget bipolar, which in turn is synaptically connected with a single midget ganglion cell. Especially in the fovea, where this system is most perfected, there are, as a rule, *neither lateral or transverse connections nor overlappings between its functional units*. The inference which may be drawn from this anatomical arrangement is that the "pure cone system" is made up exclusively of the monosynaptical structural-functional units, and all arranged parallel. The units, although close together, are nevertheless separated from their immediately adjoining neighbors by smaller or larger intervals. *This implies the ability of each unit to function independently.* Such a system of neurons may be compared with a cable composed of a great number of individually insulated wires or, more properly, chains, each chain consisting of three links, the cross section of the cable anywhere along its length giving the same, or at least a very similar and quite detailed, pattern. The essential characteristic of the pure cone system is its simplicity, as against the complexity or composite constitution of other receptor-conductor systems of the retina. It is that this strikingly simple and regular arrangement may permit a *share* of the cone excitations to pass *from the photoreceptor layer of the retina to the brain independently*, each minute, locally restricted impulse using its own "private" conductor, with no fusion with, or interference from, other similar minutely restricted nervous currents which pass along the other conductor units of the same system. There is small doubt, therefore, that the "pure, or private, cone system" of the retinal neurons is the instrument for the most delicate spatial analysis or discrimination of the visual stimuli, variously termed as "space sense" or "visual acuity."
>
> The problem of the presence or absence of the individual synapses in the nervous system, and the related question as to whether or not a "transmission" of dynamical changes across the individual synapses and along the pathways restricted to single neurons is possible, had often been, and has remained until the present, an object of debate, mostly without any anatomical observations to rely upon. In the visual system the question was whether or not a

* The stratification of the retina is defined by Polyak as follows, the numbers and letters corresponding to those at the left of Fig. 14:

1. Pigment epithelium.
2. Bacillary layer.
   - 2-*a*. Outer segments of rods (*a*) and cones (*b*).
   - 2-*b*. Inner segments of rods (*a*) and cones (*b*).
3. Outer limiting or reticular membrane.
4. Outer nuclear layer.
   - 4-*a*. Outer zone of the outer nuclear layer.
   - 4-*b*. Inner zone of the outer nuclear layer.
5. Outer plexiform layer.
   - 5-*a*. Outer zone made up of the inner fibers of rods (*a*) and cones (*b*), and of the corresponding portions of Müller's radial fibers (*u*).
   - 5-*b*. Middle zone formed chiefly by the terminal rod spherules and basal cone pedicles.
   - 5-*c*. Inner zone in which are found various bipolar cells (*d, e, f, g,* and *h*), horizontal cells (*c*), and some of the so-called "amacrine cells" (*i*).
6. Inner nuclear layer.
   - 6-*a*. Outer zone made up of the bodies of horizontal cells (*c*) together with bodies of centripetal bipolar cells (*d, e, f, g,* and *h*).
   - 6-*b*. Outer intermediate zone composed chiefly of the bodies of midget bipolars (*h*) and bodies of other centripetal bipolar varieties (*d, e, f, g*).
   - 6-*c*. Inner intermediate zone.
   - 6-*d*. The inner zone of the inner nuclear layer.
7. Inner plexiform layer.
8. Layer of ganglion cells.
9. Layer of optic nerve fibers.
10. Inner limiting membrane.

nervous excitation arising from a single photoreceptor, specifically from a single cone—provided the stimulation of a single cone was possible on physical and physiological grounds—could be transmitted to the brain as a single, isolated, independent process. Advocated by some, such an arrangement of the nerve cells and fibers was equally emphatically denied by others, to whom it seemed an extreme beyond need and possibility. . . . And yet, as shown on several occasions in this volume, there can be no doubt that in the primate retina there is a system of neurons, the "pure, or private, cone system," which in a paradigmatic way exhibits an *individual relationship on not one but two synaptical levels*, although the actual relationships, as discussed, are much more complex than heretofore thought, since besides the "one-to-one synapses" *the same neurons possess other spatially less restricted or "diffuse" relationships by means of which they are tied with the rest of a highly intricate tissue.* [The italics are ours.]

It is apparent from these quotations that Polyak is very emphatic in his statements concerning the one-to-one synaptic relationships of some of the cones, and we assume that they apply particularly to the cones clustered immediately around the exact center of the fovea, and that nerve pulses arising in a single cone are at least *in part* transmitted directly, with no admixture of nerve pulses arising in an adjacent cone, along an individual fiber of the optic nerve to the synaptic region in the lateral geniculate. At first sight this might be interpreted as denying the existence of any lateral connections in the retina between the neurons which make up the small region immediately around the fixation point where visual acuity has its maximum value. The last part of the last sentence in the final quotation, however, throws doubt upon such an interpretation since he says "the *same neurons* possess other spatially less restricted or 'diffuse' relationships by means of which they are tied with the rest of a highly intricate tissue." It is necessary, therefore, to consider the possible effect upon visual acuity of these implied lateral synaptic connections. Probably the most important of these are the horizontal or outer associational cells. These are true neurons, the bodies of which, together with their processes, are located in the outer zone, 6-*a*, of the inner nuclear layer and in the inner zone, 5-*c*, of the outer plexiform layer. These zones lie immediately adjacent to (in the vertical direction) the zone in which are located a large majority of the cone pedicles and the rod granules.

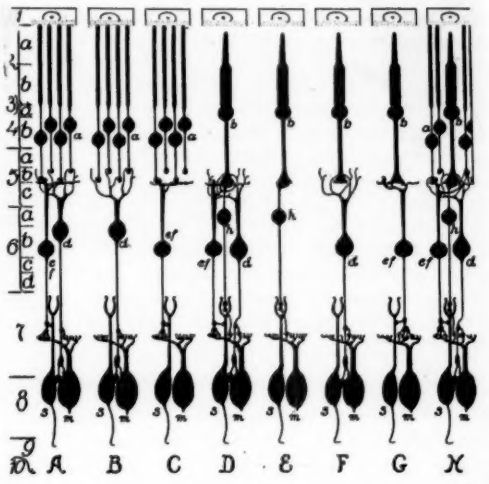


FIG. 14. Diagram reproduced with permission from "The Retina" by Polyak,[8] for which the following caption is given: "Grouping of the neurons of the primate retina into functional systems: (*A*) rod system; (*B*) rod system with only one intermediate constituent, the mop bipolar (*d*); (*C*) rod system with only the brush (*e*) or the flat bipolar (*f*) as the constituent; (*D*) cone system; (*E*) pure cone system; (*F*) cone system with the mop bipolar (*d*) as the constituent; (*G*) cone system with either the brush (*e*) or the flat bipolar (*f*) as the constituent; (*H*) mixed or common rod and cone system."

After discussing the relationships between the cones, the midget bipolars and the midget ganglion cells, Polyak comments (page 179):

There are still other nervous elements in the retina besides those already mentioned. Such are the horizontal cells, whose expansions spread horizontally in the outer plexiform layer. These are considered to be associational cells *not involved in* a direct transmission of impulses to the brain. [The italics are ours.]

There is, however, definite evidence that there are synaptic relations between the dendrites of the horizontal cells and the cone pedicles (which are the terminations of the axis cylinders of the cone neurons). To obtain a clear picture of the probable synaptic connections between the foveal cones and the horizontal cells, we must keep in mind that in the foveal region the normal position of the various retinal layers is profoundly disturbed by the almost complete absence of some of the inner layers of the retina in that region. This situation is clearly stated by Polyak (page 209):

This results in the gradual thinning of the ventral layers, beginning at the foveal edge and attaining its maximum

reduction in the flat floor (Macaque) or foveola (in Ma...) Here, in a territory measuring approximately 350μ across, practically all layers from 5-*b* vitrealward are absent. In the very center of the human foveola even layers 4 and 5 are so much reduced in thickness that in a territory measuring approximately *150μ across all layers inward from the outer limiting membrane (3), taken together, are thinner than the bacillary layer (2) alone (Fig. 38).* It is perfectly clear that this extraordinary reduction of the ventral layers in the axial region of the retina—together with the flat or almost flat floor of the primate fovea, whereby the diffraction with the concomitant dispersion of the vertically incident rays is largely eliminated—increases the sharpness of definition of physical images focused upon the outer cone segments by the dioptric apparatus of the eye, since the diffraction of the light rays during their passage through the thinned layers 4–9 is reduced to the minimum. [Italics are ours.]

According to our interpretation of Polyak's statements and of the illustrations pertaining to them, only a few of the pedicles of the cones forming the central region of the fovea are located in the foveal floor. The majority of these pedicles are attached to the ends of the long inner cone fibers and are located probably in the region immediately around the floor of the fovea. Here, as we interpret the literature, several, but not many, cone pedicles have synaptic connections with the dendrite of a single horizontal cell. The axis cylinders of the horizontal cells are, in general, long, and these spread out in all directions, being confined mostly to the inner zone of the outer plexiform layer (5-*c*).

Concerning the synaptic connections of the horizontal cell, we find this very informative statement by Polyak (page 260):

> From the connections of the dendrites, from the direction of the axis cylinders, and from the contact of the teledendric expansions it appears possible that each horizontal cell connects synaptically a small group of closely adjoining cones (doubtful whether rods also)—from which it receives impulses—with a larger group of both cones and rods in a *distant region of the retina, upon which it discharges the impulses.* Considering their morphological characteristics and their synaptical connections, there is no doubt that the horizontal cells are true nerve cells. [Italics are ours.]

If the horizontal cell is a true neuron and hence is polarized like other neurons, then neural disturbances can travel only in one direction, namely, from the dendrite to the teledendric termination of the axis cylinder. This indicates that while some of the nerve impulses generated in a central photoreceptor may be propagated from the cone pedicle through the synapse with the dendrites of the horizontal cells and outward along the axis cylinders of these cells, there is no indication that the nerve impulses are coming into the cone pedicle from the horizontal cells.

Assuming that our interpretation of this evidence concerning the role of the horizontal cells is correct, then there is little reason to believe that these synaptic relations can interfere with the functioning of the individual cones as units from the standpoint of visual acuity. The neural disturbances which arise in the horizontal cells by virtue of their synaptic relations with the cone pedicles, according to the previous quotation, travel some distance outward from the central area before they "discharge the impulses" into the neural chain of a large group consisting of both cones and rods. This cannot interfere with the visual acuity of the small area immediately around the fixation point or with the control of the visual acuity largely by the center distance between the individual photoreceptors in that region.

Aside from the cone-midget bipolar synaptic level, there seems to be but one other possibility of lateral interconnection. This is at the midget bipolar-midget ganglion cell synaptic junction. The question as to whether there are in the retina any elements which form lateral linkages between the ganglion cells and possibly other cells within the inner plexiform layer is a legitimate one. One would expect to find such inner or internal association cells either in the ganglion or in the inner nuclear layer. However, there seems to be no positive histological evidence for the existence of such lateral linkages.

Finally, the functional role of another group of cells located in the inner layers of the retina, namely, the "amacrines," must be considered. The evidence indicates that these do form certain lateral connections between different parts of the retinal structure at the ganglion cell level. The synaptic relations are not well understood and it is impossible at this time to draw any definite conclusion as to whether or not these cells function in such a way as to interfere with the individual characteristics of the central foveal cones.

Perhaps the picture which can be visualized

from the discussion thus far given represents an oversimplification of the interretinal synaptic connections. Thus, we find Polyak, after laying considerable stress on the monosynaptic relations of the central foveal cones, speaking as follows:

> The mechanism of the retina is not simple however. It is not composed only of monosynaptical relationships as already hinted not even in the center of the fovea. The individual *h-s* (i.e., midget bipolar-midget ganglion) synapse although in a sense functionally most important does not tell the whole story of the actual synaptic relationships of the midget ganglions.

He then goes on to discuss at some length other polysynaptic or diffuse synaptic relations which exist between the various types of bipolar and ganglion cells. Having thus upset to a certain extent the idea of the individualistic role of each foveal cone, he reverts again to the importance of the monosynaptic cone-bipolar-ganglion cell chain:

> The function of the *h-s* synapses is apparently not impaired by the simultaneous existence of multiple or polysynaptic relationships since the midget synapses do not overlap reciprocally.

The experimental evidence concerning the relation between visual acuity and distance from the visual axis (see Fig. 8), in our opinion, indicates the absence of any appreciable interference in the form of lateral synaptic connections, at least in the central portions of the rod-free area of the retina.

Direct Measurements of the Neural Response in the Optic Nerve

During the past two or three decades considerable work has been done on the measurement of the response occurring in the fibers of the optic nerve arising from the stimulation of the photoreceptors by light. Among the earliest of the publications on this subject may be mentioned those of Adrian and Matthews,[32] who recorded the impulses generated in the eel's optic nerve consisting of a large number of individual fibers. Similar studies were made by Granit,[33] using a mammalian optic nerve. These workers report the same general type of response consisting of a series of rapid impulses which last for a brief time, beginning a few hundredths of a second after the light is turned on. Impulses continue to be discharged at a relatively lower frequency as long as the light continues to act on the receptors. Immediately after the termination of the stimulation there is again a vigorous outburst of closely spaced impulses which die down gradually.

A little later, Hartline and Graham,[26] by using the lateral eye of the horseshoe crab, were able to isolate a single fiber of the optic nerve in which they measured the response attributed to stimulation of the eye by light. The eye of this crustacean has a relatively simple structure in that the optic nerve fiber derives directly from the photoreceptor cell without any synaptic junctions, such as are found in the vertebrate eye. The response in a single fiber of the crab's eye was found to consist of an initial relatively high frequency group of impulses which, after a few milliseconds, settle down to a relatively lower frequency series of impulses which continue at approximately a constant frequency as long as the light continues to stimulate the photoreceptor. They have established many interesting relationships between the intensity of stimulation and the frequency of impulse response, etc.

A little later Graham[34] continued the work with the eye of the horseshoe crab and elaborated the results reported in the previous reference.

Finally, Hartline[35] began work on a vertebrate eye, that of the bullfrog. In the case of the vertebrate eye it was necessary to modify the technique used previously, but he was successful in isolating single fibers and measuring the nerve pulses resulting from stimulation of the receptors by light. The retina of the frog consists largely of rods but contains some cones. It has a structure resembling in many respects that of the

[32] E. D. Adrian and R. Matthews, "The action of light on the eye. I. The discharge of impulses in the optic nerve and its relation to the electric changes in the retina." J. Physiol. **63**, 378 (1927); "II. The processes involved in retinal excitation," J. Physiol. **64**, 279 (1927–28); "III. The interaction of retinal neurones," J. Physiol. **65**, 273 (1928).

[33] R. Granit, "The components of the retinal action potential in mammals and their relation to the discharge of the optic nerve," J. Physiol. **77**, 207 (1933).

[34] C. H. Graham, "The relation of nerve response and retinal potential to number of sense cells illuminated in an eye lacking lateral connections," J. Cell. and Comp. Physiol. **2**, 295 (1932–33).

[35] H. K. Hartline, "The response of single optic nerve fibers of the vertebrate eye to illumination of the retina," Am. J. Physiol. **121**, 400 (1938).

human eye in that there is a chain of three neurons: the receptor cell, the bipolars, and the ganglion cells, the axis cylinders of which become the fibers of the optic nerve. The most striking difference between the nerve responses found in the optic nerve fibers of the frog compared with those of the horseshoe crab is that only about 20 percent of the individual fibers give a response similar to that found in all of the fibers of the crab. In these fibers, after a short latency period (a few milliseconds in duration) following the onset of illumination, a short high frequency burst of nerve pulses occurs, followed by a continuous but relatively low frequency series of impulses which persist as long as the stimulus continues and terminate abruptly with the removal of the stimulus. If, during stimulation by light, the intensity is increased or decreased, the frequency of the continuous series of pulses increases or decreases correspondingly. If, during stimulation, the intensity of the stimulus is increased by a sufficient amount and with sufficient suddenness, a high frequency burst of nerve pulses may appear, followed by a decrease to the continuous lower frequency train of pulses, as determined by the new level of excitation. On the other hand, a sudden and large decrease in the stimulation does not produce a similar group of high frequency responses, although the periodicity of the pulses does decrease to the lower level corresponding to the new lowered level of stimulation.

In about 50 percent of the fibers which Hartline examined the response was very different, consisting of a series of high frequency pulses following the application of the stimulus after a short latency period. This series is of relatively very short duration, after which there is no further response until the stimulus is removed, when a second group of high frequency pulses appears. This also is of relatively short duration. If, during the application of the stimulus, the intensity is suddenly increased, a few relatively high frequency impulses may appear and, on the other hand, if the stimulus is suddenly decreased by an appreciable amount, a short burst of responses is found, after which no further response occurs until the excitation is terminated, when the typical "off" response consisting of a few high frequency pulses again appears. This type of response is referred to as the "on-off" type.

In approximately 30 percent of the fibers no response occurs at the time of or immediately after the application of the stimulus and, in fact, no pulses are found until the illumination is discontinued, whereupon, after a very short period, a strong high frequency group of impulses of short duration occurs. With these fibers a sudden increase in the stimulation from an established level does not produce any response, but a sudden decrease in stimulation, if of sufficiently great magnitude, will produce the typical "off" response, even though the stimulation may not be entirely removed. Furthermore, if a relatively large bundle of fibers is used at one time, the response resembles very closely that previously reported by Adrian and Matthews and by Granit. The majority of the nerve fibers used by Hartline gave responses falling into one of the three types discussed. He did find, however, a few fibers which give responses differing in some details from the three predominant types. These he refers to as "freaks."

The three types of response described in the preceding paragraphs are illustrated in Fig. 15, which is reproduced (with the permission of the publisher and the author) from one of Hartline's publications.[35,36] This figure shows three oscillograph records obtained by Hartline. The caption, which is that used by Hartline, explains clearly the significance of the three records.

The question arises as to whether it is safe to extrapolate these conclusions and to say that they represent characteristically the nerve impulses traveling along the fibers of the optic nerve of the human eye. So far as we have been able to learn by perusal of the literature, no such measurements have been made on the fibers of the optic nerve of the human eye, or, in fact, of any warm-blooded vertebrate. However, concerning this situation Hartline[35] says (page 410):

Responses in other vertebrates. Responses in optic nerve fibers have been reported in a variety of cold-blooded vertebrates other than the frog. These include one shark, one *Necturus*, a number of turtles and alligators, one iguana, and several varieties of snakes. While not extensive,

[36] H. K. Hartline, "The nerve messages in the fibers of the visual pathway," J. Opt. Soc. Am. **30**, 239 (1940).

they show unmistakably the same general result that is found in the experiments on frogs. In the eyes of all these animals there are fibers which maintain a discharge as long as the retina is illuminated (or at least for the first several minutes), other fibers which give bursts of impulses only in response to changing the intensity of light, and still others which respond only when the illumination is reduced. Minor differences have, however, been found, and a more exhaustive comparative study might even reveal significant variation in optic function among different vertebrates. Thus, in the experiment on *Necturus* the maintained discharge disappeared completely after a half-minute exposure to light, and the "off" responses were feeble and required at least ten or more seconds' exposure before they could be elicited. And in both the turtle and alligator eyes fibers are not infrequently found giving only a brief burst when light is turned on, with neither maintained discharge nor "off" response. On the other hand, where one might expect to find striking differences, as in the pure cone retina of the snake, neither the actual types of response nor the relative frequency with which they were obtained differed notably from those in the mixed, though predominantly rod retina of the frog.

Hartline's results indicate that a given nerve fiber always responds with some one of the three typical modes described previously. He tried various methods of applying the stimulus and of introducing other experimental variables without being able to modify to any appreciable extent the characteristic discharge of a given fiber which he concludes is probably controlled by the functional characteristic of the ganglion cell from which the nerve fiber emerges, although he does admit the possibility that "the diversity of response might on the other hand originate in the layers of the retina between the rod and the cone layer and the ganglion cells." Hartline attempted to map out a region which he termed "the receptive field of a fiber" in the retina of the frog. He found that neural response in a given fiber could be elicited by the illumination of any part of a relatively small retinal area. This he interprets as being evidence of spatial summation indicating that a relatively large number of rods and/or cones are connected to the ganglion cell of which the axis cylinder is being studied.

From our point of view the importance of Hartline's work lies in the fact that he has definitely isolated three typical neural responses in different fibers of the optic nerve of the vertebrate eye. If it is safe to assume that this result can be extrapolated to apply to the human eye, and specifically to the small central part of the fovea for which visual acuity is maximum, then we may obtain a better concept of the type of neural responses which are responsible for the perception of graininess. We are inclined to question the applicability of Hartline's measurements on the "receptive field of the fiber" as applied to the central fovea of the human retina. While we have not found any definite statement on this subject, we are under the impression that in the frog's eye there is no well-developed area, which can be called a fovea, inhabited exclusively by cones having monosynaptic relationships to the bipolar and ganglion cells. In commenting upon the possible influence of the spatial summation which he believes to exist in the retina of the frog, Hartline accepts the possible difficulty which may arise in an attempt to apply these data to human vision. He says, "Nevertheless one can hardly expect the resolution to be as effective as in the fovea where each optic nerve fiber probably corresponds to only one receptor element." Even in the case of the receptive field as measured by Hartline, we are inclined to suspect that some other factors than spatial summation within the retina of the frog may contribute to the excessively large area which he concludes is connected to a single ganglion cell. Hartline's results strengthen the belief that in the small central area of the fovea in the human retina each receptor functions as an individual unit, but instead of each unit yielding the same type of neural response in the fibers of the optic nerve, we must conclude that there are in fact three types of response.

Reverting to our previous discussion of the type of stimulation which the retinal receptor receives when an attempt is made to fixate on a boundary line between two areas differing in luminance or on a small visual element either lighter or darker than its background, it is obvious that the receptors are being subjected to what essentially is a flickering or fluctuating stimulation. Some of these receptors within a range controlled by the amplitude of the physiological nystagmus are subjected to a rapidly fluctuating illuminance which can be expected to result in the off-and-on response of about 50 percent of the receptors involved. Others, receptors outside of the range of the nystagmic

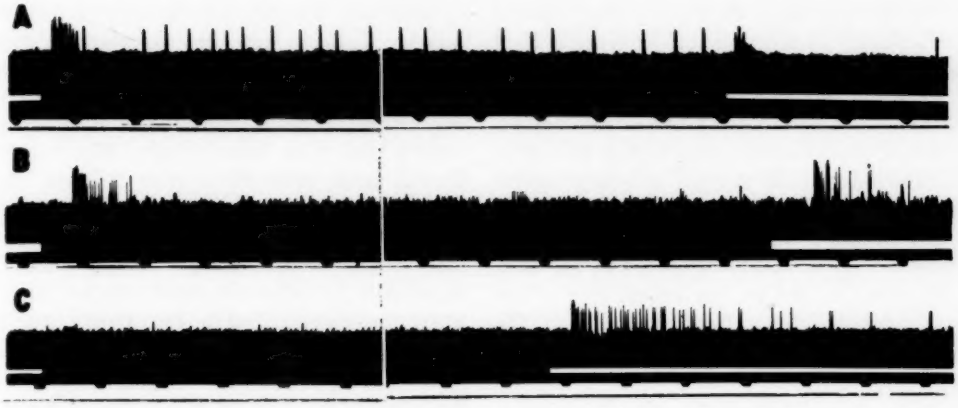


FIG. 15. Oscillograph records reproduced with permission from Hartline[37] for which he gives the following caption: "Oscillographic records of the action potentials in three intraocular optic nerve fibers of the frog's eye, showing three characteristic response types."

amplitude, are subjected to a continuous illumination of different levels, and in these regions about 20 percent of the receptors are responding by nerve pulses of approximately uniform frequency, the frequencies for the two fields being different by an amount depending upon the magnitude in the difference of their luminances. Another group of nerve fibers coming from receptors within the zone of variable illuminance are responding with the typical off-response. On being conducted into the cortex, this complicated barrage of neural response must finally in some complex way build up a perception of a sharp dividing line between two parts of the visual field or of the presence of a light or dark spot on a field of uniform brightness. When we attempt to extend this to what is happening when a judgment of blending distance is being made, the problem becomes so complex that it is difficult of comprehension. However, we believe that these direct data concerning the neural response arising from the stimulation of the photoreceptors again support the idea that the effective size of the visual field element which is important is that corresponding to the size of or the separation between two adjacent cones at or very close to the exact center of fixation.

Indirect or Sensory Evidence

There is a voluminous literature concerned with the sensory or perceptual evaluation of the various physical aspects of the visual stimulus. From results of this type of work many conclusions have been drawn concerning the various modes of functioning of the visual apparatus, including, for instance, the probability of interaction within the retina itself between the variously stimulated elements of the retinal structure or in variously stimulated selected areas of the retina. Many of the conclusions drawn from this type of sensory or perceptual experiments are in our opinion not very convincing. It is certainly very difficult to interpret some of the results of this type of work and, in fact, a different interpretation can be placed on the same group of experimental data by different individuals.

We shall not attempt a comprehensive survey of this material, since it is entirely too voluminous to be discussed at this time. As illustrative, however, of this general type of data may be mentioned some of the work of Granit and his collaborators.[37–39] These authors have determined

[37] R. Granit and W. A. Davis, "Comparative studies on the peripheral and central retina. IV. Temporal summation of subliminal visual stimuli and the time course of the excitatory after-effect," Am. J. Physiol. **98**, 644 (1931).

[38] R. Granit and E. L. Hammond, "Comparative studies on the peripheral and central retina. V. The sensation-time curve and the time course of the fusion frequency of intermittent stimulation," Am. J. Physiol. **98**, 654 (1931).

[39] C. H. Graham and R. Granit, "Comparative studies of the peripheral and central retina. VI. Inhibition, summation, and synchronization of impulses in the retina," Am. J. Physiol. **98**, 664 (1931).

the fusion frequency of intermittent stimuli under a variety of experimental conditions. They conclude that there is definite evidence of the temporal summation of subliminal visual stimuli.[37] They also conclude from their experiments that there is interaction within the retina itself between stimulated areas and they suggest that this interconnection may occur at the cone (or rod)-bipolar synaptic level and, further, that possibly this interlinkage is provided by the horizontal cells. It should be remembered that Polyak's conclusion on this point is that the horizontal cells do not interfere with the effectiveness of monosynaptic relationships between the cone and the midget bipolar. Granit[38,39] and his co-workers have also interpreted some of their experimental evidence to indicate that there is a mechanism within the retina itself which, under certain conditions, exerts an inhibitory effect on the spatial summation mentioned. This inhibition appears, according to their results, to be of greatest importance in the foveal region and manifests itself only when there is differential stimulation on two parts of the retina more or less closely related spatially. They suggest that possibly the amacrine cells may be responsible for this inhibitory effect. It will be recalled that Polyak expressed the opinion that there is very little histological or anatomical evidence concerning the synaptic relation of amacrines with other neurons in the retina, and hence from such evidence it is at present impossible even to guess as to the possible functional role of these retinal elements. The interpretation placed upon their experimental results by Granit and his collaborators may, therefore, indicate the possibility of spatial summation within the retina itself and would be expected to result in a depressed visual acuity. This may be entirely reasonable if applied to the extra-foveal parts of the retina where the histological evidence indicates that more than one, and in some areas several, cones are connected with a single fiber of the optic nerve. However, the histological evidence and the relation between visual acuity and angular distance from the fixation point (Fig. 8) lead us to doubt seriously the existence of spatial summation in the central part of the fovea centralis. In this particular region visual acuity and distance between centers of adjacent cones, *CD*, march hand in hand and it appears doubtful that this could happen if there were any appreciable spatial summation. Nor can the existence of an inhibitory action upon spatial summation when differential stimulation occurs be appealed to as an explanation of the high visual acuity found on the fixation axis. While it is true that the visual acuity test objects usually used in this kind of work have high objective contrast, the distribution of illuminance in the retinal image (as determined by diffraction and nystagmic motion) when the elements of a test object are just resolved, produces relatively low illuminance gradients on the retina. It is difficult to believe that these small differences in retinal stimulation could possibly evoke sufficient inhibitory effect to counteract the spatial summation.

In conclusion we do not find in such sensory or perceptual experiments convincing proof of the existence of spatial summation within that part of the retina (the central fovea) which is determinative of blending distance in the making of graininess measurements. We, therefore, adhere to the opinion previously expressed that for our purpose the effective size of the retinal unit must be considered to be that of the diameter of the central retinal cones or, more properly, the distance between centers of such adjacent elements.

DISCUSSION

Since the evidence just given concerning the mode of functioning of the human visual system throws some new light on the granularity-graininess problem, we shall return now to a further consideration of possible techniques of making granularity measurements and of methods for interpreting them in such a way as to yield the desired correlation between granularity and graininess values.

Before proceeding with this discussion, it seems desirable to establish beyond any doubt the exact meaning of the terms *transmittance* and *density* which we must use so frequently in referring to the amount of radiant energy or light absorbed by relatively large or very small samples of a developed silver deposit. When a sample is very large compared with the developed silver grains, the term *transmittance* as usually used actually means the *average* transmittance, $\bar{T}$, of the area

measured. Average transmittance is defined as

$$\bar{T} = (T_1 + T_2 + T_3 + \cdots T_n)/n,$$

where T_1, T_2, T_3, etc., are the transmittances of small surface elements, of equal areas, of the developed silver deposit. The density of such a sample is given by the familiar equation,

$$D = \log_{10} 1/\bar{T}.$$

In the case of large samples we sometimes forget that the transmittance or density from ***point*** to ***point*** on the sample is actually variable.

In the measurement of granularity it is frequently necessary to measure the amount of radiant energy or light transmitted by very small elements of the developed silver deposit and in many cases these areal elements may vary, from point to point, from very low to very high transmittance. Here, it is obvious that the transmittance (or density) is not uniform over the area being measured. The value of transmittance assigned to such a small surface element is again that of average transmittance, $\bar{T}$, and density is again expressed in terms of the log of the reciprocal of this average transmittance. It is incorrect to apply the term "average density" to this value, and while the term "integrated" is not entirely appropriate, we shall use this word as a modifier to indicate that density values applying to small surface elements are not uniform and are not *average* densities. Integrated density, ID, is therefore defined as

$$ID = \log 1/\bar{T}.$$

We wish to call attention again to some of the data published in one of our previous communications.[1] In the fourth column of Table VII are given values (CD_{vf}) of the distance between centers of adjacent retinal receptors (foveal cones) when projected by the dioptrical elements of the eye onto the visual field (vf) at the blending distance. Since the magnification in the graininess instrument is 32, the distance between centers of the cones referred back to the photographic deposit, p, itself is obtained by dividing the values in column 4 by 32. These values (CD_p) are shown in the last column of the table. They may be considered to correspond quite closely to the diameters of the scanning apertures which must be used in any case if it is desired to determine the difference in average transmittance, $\bar{T}$ (or integrated density), between the areas on the developed silver deposit corresponding in size to the individual cones in the central portion of the fovea when the sample is viewed at the blending distance, or when its magnified image (at any value of magnification) is viewed at the blending distance. This latter statement assumes that the blending distance is directly proportional to the magnification used in any case. This assumption is not strictly valid, but for the purposes of the present discussion the departures from proportionality between blending distance and magnification may be neglected.

In Table VII[1] data are given for four photographic materials varying widely with respect to the coarseness of the granular structure and for a series of densities (expressed in terms of diffuse density) varying from relatively low to high values. It will be seen that for the Tri-X material the diameters of scanning apertures corresponding to an individual cone vary from 9.2 to 14.9μ, while for the material having the finest granular structure, F.G. Neg., these values lie between 2.9 and 3.2μ. It is evident, therefore, that if a technique is to be employed in the measurements of granularity which will give the frequency of occurrence of densities, ID, for areal elements corresponding in size to the size of the retinal receptors, the diameter of the scanning aperture must change over an appreciable range when dealing with different materials. Although this does not appear to be a very attractive procedure from the instrumental point of view, nevertheless if this is required in order to obtain the desired correlation between granularity and graininess measurements, it is a necessity which must be faced.

By using the data in Table VII it is possible to make up composite photomicrographs which represent the configuration of the illuminance distribution on the retina when an observer is looking at a granular pattern at the blending distance. Four such pictures are shown in Fig. 27.[1] The hexagonal grid of white lines represents a conventionalized mosaic of closely packed hexagonal cones. The magnification of the grain pattern superposed on these hexagonal grids is adjusted in each case to such a value, determined by the blending distances shown in Table VII,

that these photomicrographs represent the pattern of illuminance in the image of the granular structure on the retinal mosaic when the granular structure, or a magnified image thereof, is at the blending distance. The structure of the illuminance pattern incident upon each retinal element is substantially similar for the four materials even though, as shown by the data in Table VII, they differ greatly with respect to the fineness of their granular structures.

These composite photomicrographs represent a static condition which can exist only during perfect fixation. It is generally agreed that perfect fixation, even for an extremely short time interval, is practically impossible. If we now imagine the retinal mosaic moving rapidly, in a manner dictated by the physiological nystagmus, with respect to the illuminance pattern incident on the retina, each retinal cone is subjected to a rapidly changing stimulus. These rapid changes in illuminance initiate neural responses which are propagated, at least for cones in the central part of the fovea, through the monosynaptic connections between cone and midget bipolar and between the midget bipolar and midget ganglion cell, thence along an individual fiber of the optic nerve.

Hartline[36,37] has shown that the neural responses measured in individual fibers of the optic nerve are not all alike but fall into three distinct groups, as illustrated in Fig. 15, where *A* represents the "on" plus continuous (relatively low frequency) response found in only 20 percent of the nerve fibers; *B* represents the "on-off" response found in 50 percent of the nerve fibers; and *C* represents the "off" response found in 30 percent of the fibers. When an observer is looking at the central portion of a large, uniformly illuminated, visual field, it follows from Hartline's results that only the cones and fibers of the optic nerve belonging to the group *A* (Fig. 15) are active and responsible for the observer's perception of brightness. Since the other two groups of cones and nerve fibers are not receiving illuminance which is changing in intensity, they cannot be generating neural responses, even though the physiological nystagmus is still active in causing movement of the retinal receptors relative to the image incident thereon. However, as soon as the attention is fixed on a visual field in which texture, graininess, or some other type of luminance discontinuity is present, the receptor groups, *A* and *B*, come into action and it appears almost certain that it must be the neural responses of these two groups of receptors which are largely responsible for the perception of the luminance inhomogeneity.

If our conclusions from the literature are correct, the total number of visual receptors responsible for the judgment of threshold graininess in the visual field is relatively small, not exceeding at most a few hundred. Since the illuminance patterns illustrated in Fig. 27[1] do produce threshold sensations of graininess, it may be concluded that these particular distributions of illuminance must be sufficient to excite in the few hundred cones clustered around the center of the fovea a sum total of neural responses which are just supraliminal.

Figure 28(a)[1] represents the configuration of the illuminance pattern on the retina when the viewing distance is decreased by 25 percent of the blending distance, the configuration for which is given in Fig. 28(b). Figure 28(c) represents the illuminance pattern when the blending distance is increased by 25 percent of the blending distance for this particular silver deposit. This decrease and increase of viewing distance is sufficient to produce, on the one hand, a very marked increase in the graininess perceptible in the visual field and, on the other, the complete disappearance of graininess. A careful visual inspection of the photomicrographs shown in Figs. 27 and 28[1] and others of similar character, suggest that possibly the just-perceptibility of graininess in a visual field may depend more directly upon the differences in illuminance incident upon *adjacent* retinal elements rather than upon the illuminance differences computed from the mean illuminance incident on the receptor field in the immediate vicinity of the center of the central fovea. This is equivalent to saying that in the measurement of granularity the determination of the frequency of occurrence of transmittance (or density) differences between a large number of *adjacent* pairs of field elements corresponding in size to the retinal receptors may be more significant as an indication of graininess than the determination of the frequency of occurrence of the individual transmittance (or

density) differences, computed from the arithmetic mean of all the measured values.

We believe that the method we are proposing for the use of the difference between the integrated densities (or average transmittances) of two small adjacent surface elements of the developed silver deposit constitutes a new procedure for the evaluation of granularity. It is desirable, therefore, to distinguish clearly between this particular density (or transmittance) difference and the differences which have been frequently used in past work, these being based upon departures from the integrated density or the average transmittance of the whole sample. For this purpose we wish to introduce a unique term, one not having an established connotation in the field, and have chosen the adjective syzygetic.* We shall continue to use the usual symbols ΔD and ΔT in referring to differences in integrated density or average transmittance computed from the mean of the sample as a whole. Thus

$$\Delta D = ID_a - \bar{D},$$

where ID_a is the integrated density of a small surface element and $\bar{D}$ is the integrated density of the whole sample, and

$$\Delta T = T_a - \bar{T},$$

where T_a is the average transmittance of a small surface element and $\bar{T}$ is the average transmittance of the whole sample. To indicate differences which apply to adjacent surface elements, we shall use the symbols $S\Delta D$ and $S\Delta T$, where these are defined by the following relationships:

$$S\Delta D = ID_a - ID_{a1},$$

where the subscripts a and $a1$ represent a pair of juxtaposed surface elements. Such a difference we shall refer to as a *syzygetic density difference.* In the case of transmittance,

$$S\Delta T = T_a - T_{a1},$$

where the subscripts a and $a1$ denote a pair of juxtaposed surface elements. Such a transmittance difference we shall refer to as a *syzygetic transmittance difference.*

Microdensitometric traces, such as are shown in Fig. 16, give a more quantitative idea of the conditions illustrated in Fig. 28.[1] These traces were obtained by using a developed sample of the Tri-X film with a diffuse integrated density of 0.43. An image of the sample when projected at a magnification of 32 times onto the screen of the Jones-Deisch graininess instrument gave a blending distance of 548 cm. A central foveal cone (diameter 1.5μ) projected by the dioptrical elements of the normal human eye onto the screen of the graininess instrument at a distance of 548 cm has a diameter of 477μ which, referred back to the silver deposit, is equivalent to 15μ (actually 14.9μ). If the size of the scanning aperture in the microdensitometer is made equivalent to a circular area on the sample having a diameter of 15μ, the trace shown at *B* in Fig. 16 is obtained. The ordinate at any point on this curve gives the integrated density, *ID* (specular), of a circular area on the sample having a diameter of 15μ. The oppositely hatched rectangular areas just below the *B* trace in Fig. 16 represent a "row" of retinal receptors. The vertical lines from the center of each of these rectangular areas where they intersect the curve give the values of integrated density, *ID*, for a series of adjacent circular areas on the sample, each 15μ in diameter, and the difference between any pair of such adjacent values gives a value of *syzygetic density difference* $S\Delta D$. The curve in Fig. 16B represents only a small part of the densitometer trace which was made and from which a large number of $S\Delta D$'s were obtained, a sufficiently large number, in fact, to determine approximately the frequency of occurrences of the various values of $S\Delta D$.

The logarithm of the average illuminance, $\log\bar{I}$, incident on each retinal receptor is inversely proportional to the integrated density of the corresponding area on the developed silver deposit. If a cone moves relative to the image on the retina, the trace *B* in Fig. 16 shows the magnitude of the change in $\log\bar{I}$ incident on that receptor;

* According to Webster's International Dictionary, Standard edition (1934), the word *syzygy* is derived from the late Latin word *syzygia* which denotes a joining together or conjunction. The word syzygy has usage in certain specialized fields and in general is applied to the close relationship or joining together of a *pair* of elements. The synonyms given in Roget's *Thesaurus* (Crowell Company, New York, 1939) assist further in establishing an understanding of the general connotation of the word syzygy from which the adjective syzygetic is derived. From page 76, section 199—contiguity, we find the following: juxtaposition, touching, syzygy, conjunction, and continuousness.

and, furthermore, if the velocity of the cone movement is known, the *time rate of change* in $\log\bar{I}$ can be determined from the gradients of the trace.

When the viewing distance in the graininess instrument is increased by an amount such that the graininess has completely vanished from the visual field, the size of the area on the projected image of the silver deposit corresponding to a retinal receptor increases, and if the viewing distance applying to the condition shown in Fig. 16B is increased by 33 percent, the equivalent cone area becomes 20μ in diameter instead of 15. By using a scanning aperture in the microdensitometer corresponding to 20μ on the sample, the trace in Fig. 16C was obtained. Here, again, the relation of the retinal cones to the variation in integrated density, ID, in the sample, or to log average illuminance, $\log\bar{I}$, incident on the individual cones, is shown by the oppositely hatched rectangular areas just below C.

In a similar manner, by making the viewing distance 33 percent less than the blending distance, an area on the sample corresponding to a retinal cone decreases to 10μ in diameter, and with a scanning aperture in the densitometer corresponding to 10μ on the silver deposit being examined, the trace at A in Fig. 16 was obtained. These three traces represent approximately the conditions illustrated graphically in Fig. 28 of our previous communication.[1] From these traces, however, we can derive some numerical values indicating the *magnitudes* of the differences which exist between the stimuli resulting in just perceptible graininess, very perceptible graininess, and no perceptible graininess.

The trace in the lower part of Fig. 16, at D, shows the relationship existing between the retinal mosaic and the illuminance pattern incident thereon when the magnified image of the same developed silver deposit is viewed at a distance (5.3 times the blending distance) such that a retinal receptor projected thereon has a diameter of 80μ. This trace was obtained by using a scanning aperture in the microdensitometer having a diameter, referred to the sample, of 80μ. The ordinate at any point on this curve therefore gives the value of integrated density, ID, of an area on the sample having a diameter of 80μ. Since at the blending distance the equivalent cone diameter is 15μ on the sample, it follows that the ordinate of the trace D gives values of the integrated log illuminance (reciprocally) incident on an area of the retina including approximately 26 cones. It is extremely doubtful that the values of either ΔD or $S\Delta D$ derived from this trace can yield much significant information for the prediction of graininess.

Figure 16 represents static conditions which do not exist. We must consider that the retinal mosaic is moving relatively rapidly with respect to the illuminance pattern. The magnitude of this motion is represented by the two horizontal lines terminated by arrow heads immediately under the B section of Fig. 16. The shorter of these two lines represents the amplitude of high frequency physiological nystagmus, while the longer one represents the amplitude of a lower frequency nystagmic motion.* In case of the high frequency nystagmus, a single cone, such as a, moves back and forth with respect to the image of the sample formed on the retinal surface by the dioptrical element of the eye, between the positions represented by the dotted rectangles a_1 and a_2, while the lower frequency nystagmic motion causes this same cone to vibrate between the positions indicated by the dotted rectangles, a_3 and a_4. It should be remembered that the four traces shown in Fig. 16 represent equal linear distances on the retina but very different distances as measured upon the sample being studied. The D trace in this figure corresponds to a distance of 1360μ on the sample, the C trace to 340μ, the B trace to 255μ, and the A trace to 170μ.

The amplitudes of the integrated density, ID,

* The lowest frequency of nystagmic motion observed by Adler and Fliegelman has a period of from 2 to 4 seconds, probably averaging around 2 seconds. The average amplitude is approximately 30 minutes. This motion is a relatively slow lateral drift of the visual axis such that the geometrical image of the vertical bright line on which the observer attempts to maintain fixation moves slowly about 15 minutes to one side of the center of the fovea then, suddenly, in a very short time, moves back to a point about 15 minutes to the other side of the foveal center. From our data (Fig. 8) the acuity at 15 minutes from the center of the fovea is only 65 percent of that at the exact center of the fovea. As suggested by Weymouth and his co-workers (reference 28) and Javal (reference 29) this low frequency nystagmic motion is probably a part of the visual system for the maintenance of fixation, so that an image of a detail in the visual field which an observer is consciously trying to see is maintained at the point on the retina having maximum acuity, that is, the center of the fovea centralis. It does not seem likely that this particular aspect of the nystagmic motion has much direct bearing on the perception of graininess other than its indirect function of maintaining approximate fixation.

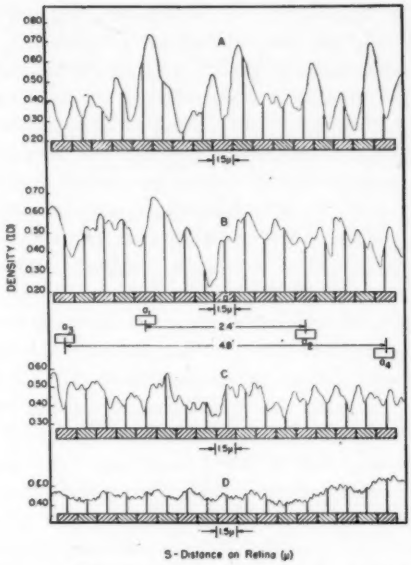


FIG. 16. Microdensitometric traces showing characteristics of visual stimuli which evoke (*A*) very perceptible graininess, (*B*) just perceptible graininess, and (*C*) imperceptible graininess.

found in the sample, and hence the amplitudes of $\log I$ incident on the receptors, are relatively high under the conditions represented by Trace *A*, intermediate for Trace *B*, lower for Trace *C*, and much lower for Trace *D*. We know that Trace *B* represents the graininess threshold, and hence we can evaluate quantitatively, at least in relative terms, the distribution of illuminance on the retina which is responsible for the just-perceptibility of graininess.

We can also obtain some quantitative values of the syzygetic density differences ($S\Delta D$) in the sample which produce luminance patterns on the retina corresponding to very perceptible graininess, just perceptible graininess, and imperceptible graininess. In Table IV, second column, are the values of maximum $S\Delta D$ for the four traces in Fig. 16, and in the third column are values of the average $S\Delta D$.

For a given change in the illuminance incident upon the retinal cone, it is generally agreed that the magnitude of the neural response depends upon the suddenness (i.e., the time rate of change) with which the change occurs. It may be profitable, therefore, to examine the gradients of the traces shown in Fig. 16. By measuring the slope at closely spaced points (0.5-μ intervals on the abscissa scale) the average gradient, $\bar{G}_s$, ($G_s = dID/ds$)† for each of the four traces has been determined. These values are shown in the fourth column of Table IV. The variation in average gradient is relatively great and appears to be in agreement, at least qualitatively, with the large difference in the appearance of the developed silver deposit when viewed under the conditions corresponding to the viewing conditions which these traces represent.

The average gradients, $\bar{G}_s$, shown in Table IV, are the rates of change of density in the sample with respect to the equivalent distance measured on the retina. They, therefore, represent spatial gradients. It is the time rates of change, however, which doubtless are the most direct determinants of neural response and these can be computed by assuming certain characteristics of the nystagmic movement. As noted previously, it appears from the work of Adler and Fliegelman that there are two typical nystagmic motions differing with respect to frequency and amplitude. However, it turns out that the velocities in the two cases are probably not very different because the movement of greater amplitude occurs at a lower frequency than the high frequency motion. If we assume that this motion is of the simple harmonic type, and this seems to be a reasonable assumption, the maximum velocity with which a retinal receptor moves with respect to the illuminance pattern is approximately 1μ per millisecond, and the average velocity is approximately 0.6μ per millisecond. The values of spatial gradients shown in the fourth column of Table IV may, therefore, be converted to time rates of change, $G_t = dID/dt$,** and if we use the average velocity mentioned in the previous sentence, the values of $\bar{G}_t$ are those shown in the last column of the table. These now are the time rates of change which may be expressed either in terms of density on the sample or in terms of log average illuminance ($\log \bar{I}$) incident upon the retinal receptors. As expressed using the time interval of milliseconds, these gradients appear to be relatively low, and perhaps when the rates of response and recovery of the retinal receptors are considered, it may be more significant to express

† s, the distance on the retina, is expressed in microns, μ.
** t (time) is expressed in milliseconds.

these time rates of change in log intensity using 0.01 second as the unit of time rather than 1 millisecond. Thus, for instance, in the case of Trace *B* in Fig. 16, the value of $\bar{G}_t$ indicates that the log illuminance on a retinal receptor is changing by 0.7 in 0.01 second. Converting this to illuminance values, it means that the average illuminance changes fourfold in 0.01 second (10 milliseconds).

Hence, when an observer is looking at a visual field in which graininess is perceptible, the retinal cones near the point of fixation are being subjected to a rapidly fluctuating illuminance. It may be of interest to determine the frequency of this fluctuation. This can be done, at least approximately, for a specified case. Let us consider, for instance, a developed silver deposit having a density of 0.3. In general, this density shows greater graininess than either higher or lower densities on that particular material. Such a density transmits 50 percent of the incident light. Approximately one-half of the area is covered by substantially opaque silver grains or clumps thereof. Moreover, these opaque grains or closely clustered clumps are roughly circular in shape and the interspaces which have higher transmittances have roundish contours. When a magnified image of such a sample is viewed at a distance such that graininess is at or slightly above the threshold, a case illustrated at *B* in Fig. 16, the sizes of the areas of low average transmittance correspond fairly well with the size of the retinal cones projected on the graininess field. Also, the sizes of the areas having relatively high average transmittance correspond approximately with the equivalent size of the retinal receptors. Now, when a cone moves with respect to the image of such a deposit formed on the retina, it requires about 2.5 milliseconds to move a distance of 1.5μ, the distance between the centers of adjacent cones, and, on the basis of the assumptions made above, 2.5 milliseconds becomes the time between maximum and minimum average illuminance. An additional 2.5 milliseconds of the same motion are required for that cone to reach a position where the illuminance incident thereon is equal to that at the beginning of this cycle. For this particular example, then, the cone is being subjected to a fluctuating illuminance having a periodicity of 200 cycles per second. As shown in Table IV, the average amplitude of this fluctuating illuminance, expressed in terms of $\log \bar{I}$, is 0.12. If the magnification is increased and the viewing distance maintained, the sizes of the substantially opaque and transparent areas in the deposit become greater with respect to the retinal receptors, with the result that the illuminance amplitudes increase while the frequency of the intermittency tends to decrease. On the other hand, if the magnification is decreased, the viewing distance remaining constant, the opposite effect occurs, namely, the sizes of the substantially opaque and transparent areas tend to become less than those of the retinal receptors, with the result that the illuminance amplitude decreases while the frequency of illuminance change tends to increase. This latter case is one resulting in the imperceptibility of the graininess pattern. We are unable at the present moment to quantify these considerations further, but they do yield some general information concerning retinal stimuli which result in the perceptibility or imperceptibility of graininess of a sample having a density of approximately 0.3.

The question may be raised as to whether or not the syzygetic differences, either of density or transmittance, can be expected to yield more or different information than that which can be extracted from the frequency of occurrence of the differences computed from the arithmetic mean of all of the measurements. Under certain conditions the two evaluations of these differences are proportionately related. Thus, if the frequency of occurrence of ΔD as measured with a scanning spot of a given size is represented precisely by the Gaussian function, then the frequency of occurrence of $S\Delta D$ can be derived therefrom and obviously no new or additional information is obtained. However, if the frequency-ΔD curve departs appreciably from the Gaussian distribution, the $S\Delta D$-frequency function cannot, in general, be derived therefrom.

TABLE IV. Amplitude and gradient characteristics of the visual stimuli represented graphically in Fig. 16.

Diameter of aperture	Max. $S\Delta D$	Mean $S\Delta D$	$\bar{G}_s$	$\bar{G}_t$
10	0.42	0.16	0.22	0.13
15	.28	.12	.12	.07
20	.21	.10	.08	.05
80	.08	.03	.03	.02

Under these conditions the experimental determination of a $S\Delta D$-frequency function does provide new and additional information. As shown in a previous communication[1] we have found that the frequency of occurrence-density distribution curve departs appreciably from the Gaussian type, especially when the smaller scanning apertures are used.

As we have pointed out previously (reference 1, page 220), for the purpose of interpreting the values of ΔD or ΔT in terms of graininess, it is useful to express the data in terms of the frequency of occurrence of these differences as a function of their magnitudes without regard to the algebraic sign. In the case of syzygetic differences, $S\Delta D$ or $S\Delta T$, the plus or minus signs disappear from the picture since algebraic signs can only be assigned by use of some artificial convention concerning the geometric positions of the two adjacent areas.

The group of curves in Figs. 17 to 22, inclusive, illustrate the frequency with which these two aspects of density differences are encountered when apertures of different sizes are used.* The data in these figures are derived from a developed sample of Tri-X material having a diffuse density of 0.43. Microdensitometer traces similar to those in Fig. 16 were made with scanning apertures equivalent to circular areas on the sample varying from 5.3μ to 158μ in diameter. From each of these traces a large number of readings of ΔD and $S\Delta D$ were made. This involved the long and laborious process of superposing the traces over grids having the proper spacings, and reading manually the density at points separated by distances equivalent to the scanning aperture diameter. From these readings cumulative frequency curves were plotted as previously described (reference 1, page 213). The frequency-density difference curves, which are the derivatives of the cumulative-frequency functions, were obtained by graphic differentiation. By examining Figs. 17 to 22, inclusive, it is seen that the frequency of occurrence of the $S\Delta D$ differs markedly from that of ΔD. In general, ΔD occurs with greater frequency than $S\Delta D$ in the region of low density differences, while in the region of higher density differences $S\Delta D$ occurs more frequently than ΔD. This is particularly noticeable in the case of the scanning apertures equivalent to the small areas on the sample, Figs. 20–22. It should be remembered that at the graininess threshold for this photographic material a retinal cone is equivalent in size to an area on the sample having a diameter of 15μ which lies half-way between the apertures used in obtaining Figs. 20 and 21. It appears, therefore, that at the threshold of graininess the syzygetic density difference has a somewhat greater frequency of occurrence in the region of ΔD's lying between 0.1 and 0.4. It is quite apparent from the data contained in Figs. 17 to 22, inclusive, that for this photographic material there is no systematic relation between the frequency of occurrence of the two aspects of density difference being considered. It can be concluded, therefore, that the use of $S\Delta D$ (or $S\Delta T$) may yield some information concerning the graininess of developed photographic deposits which cannot be obtained from the measurement of ΔD.

It may be of interest to compare the averages of the syzygetic density differences, derived from the curves in Figs. 17 to 22, inclusive, with those of the density difference computed from the mean. These are shown in the second and third columns, respectively, of Table V. The average of $S\Delta D$ is somewhat larger than that of ΔD in all cases except for the values determined by the use of the scanning aperture equivalent to 158μ on the sample. The ratios are shown in the last column and it will be seen that these decrease as the size of the scanning aperture increases. For this material at least there is little significant difference in the ratios obtained with areas having diameters from 5 to 40μ, inclusive. However, there is no evidence to indicate that this ratio will be the same or constant for the same range of scanning apertures for other photographic materials.

A striking example of the difference in shape of the ΔD and $S\Delta D$ frequency curves is found in the case of two photographic materials, A and B. These materials have approximately the same speed, but one of them, A, was found by practical test to be very inferior to B from the standpoint of graininess. Projection prints can be made from the material B with a considerably greater

* To illustrate this point we have chosen to show the results in terms of *density* differences. This must not be interpreted as meaning that we have as yet made a decision that differences expressed in terms of density are more directly significant of gammas than the same differences expressed in terms of transmittance.

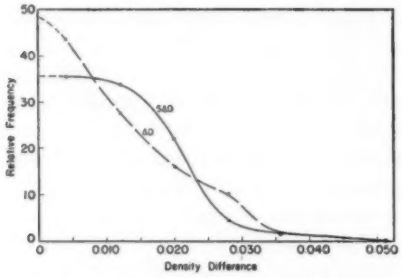


FIG. 17. Curves showing the relation between relative frequencies of occurrence of $S\Delta D$ and ΔD obtained by using a scanning aperture equivalent to a diameter of 158μ on the sample.

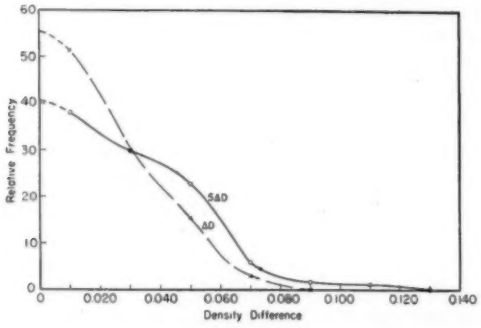


FIG. 18. Curves showing the relation between the relative frequencies of occurrence of $S\Delta D$ and ΔD obtained by using a scanning aperture equivalent to a diameter of 79.8μ on the sample.

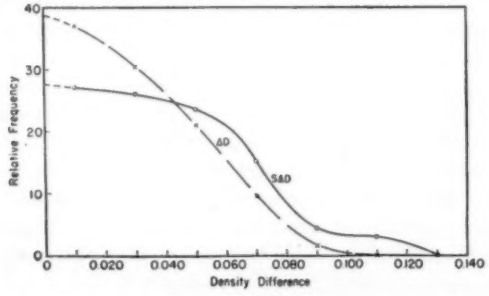


FIG. 19. Curves showing the relation between relative frequencies of occurrence of $S\Delta D$ and ΔD obtained by using a scanning aperture equivalent to a diameter of 39.9μ on the sample.

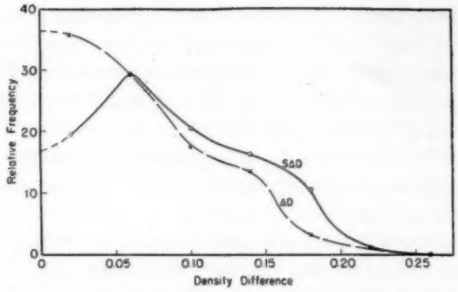


FIG. 20. Curves showing the relation between the relative frequencies of occurrence of $S\Delta D$ and ΔD obtained by using a scanning aperture equivalent to a diameter of 20.2μ on the sample.

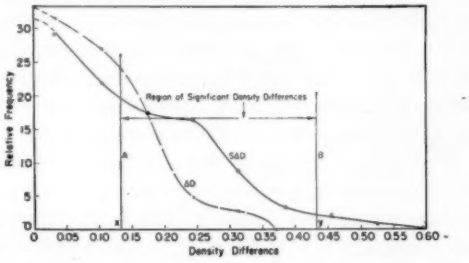


FIG. 21. Curves showing the relation between relative frequencies of occurrence of $S\Delta D$ and ΔD obtained by using a scanning aperture equivalent to a diameter of 10.7μ on the sample. This figure also illustrates the isolation of the region of significant density differences.

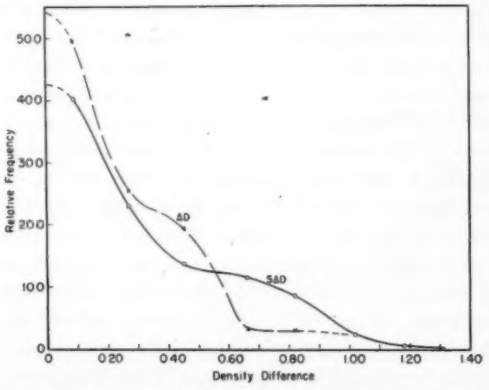


FIG. 22. Curves showing the relation between relative frequencies of occurrence of $S\Delta D$ and ΔD obtained by using a scanning aperture equivalent to a diameter of 5.3μ on the sample.

enlargement than from material A, before objectionable graininess is encountered. The distribution curves for ΔD, measured with a scanning aperture equivalent to an area 80μ in diameter, on these two materials are shown in Fig. 23. The root-mean-square averages are practically identical and an inspection of the two curves shows little if any difference, although it might be concluded that material B would be slightly inferior to A with respect to graininess. By reducing the diameter of the scanning aperture to the equivalent of 5.3μ on the sample, the distri-

bution curves of ΔD in Fig. 24 were obtained. Here, again, the root-mean-square averages are very nearly the same, although there does appear to be a tendency of the material A to give somewhat higher frequencies of occurrence of ΔD for values greater than 0.10. This might be interpreted as indicating somewhat greater graininess for the material A, although admittedly the evidence is by no means conclusive. The distribution curves for $S\Delta D$, obtained by again using a scanning aperture of 5.3μ, are shown in Fig. 25 and here the result seems to be quite in line with the practical experimental results. The material A now shows a secondary maximum at $S\Delta D = 0.13$ with a few values extending out beyond 0.30. In the case of these two materials, it is the presence of relatively few large values of $S\Delta D$ which results in the higher graininess found under practical conditions.

We wish to reiterate that in our opinion it is unlikely that any average (arithmetic, root-mean-square, etc.) of *all* the observed density or transmittance differences in any particular case will serve as a significant index of graininess and that probably the desired correlation between objective measurements and graininess will involve the use of some characteristics, Z, of the syzygetic differences lying between certain limits, x and y. To be useful in the measurement of granularity these limits must of necessity be the same for all samples having a given density value, even though such samples are derived from any of the photographic materials, either negative or positive, which are to be viewed by transmitted light (either directly or by projection). Preferably the values of x and y should be the same for all densities, but this is not a necessary requirement for usefulness. The location of what may be termed the "region of significant density differences" is illustrated in Fig. 21 where the vertical lines A and B are drawn through the hypothetical limiting values, x and y, of $S\Delta D$. The hypothetical characteristic of such a limited group of syzygetic density differences may be defined formally as $Z = f(S\Delta D_x \text{ to } S\Delta D_y)$, where x and y represent the minimum and maximum values of $S\Delta D$ used in evaluating Z. There are several possible courses which may be followed in establishing a correlation between values of Z and the corresponding graininess values, of which, for purposes of illustration, we shall mention some of those that appear most promising.

Before doing this we shall digress to discuss in some detail certain aspects of visual stimuli which give rise to the perception of threshold graininess when a series of samples, all from the same photographic material, but differing in transmittance, are viewed by an observer. For the sake of simplification we shall use for this purpose a series of conventionalized patterns simulating developed silver deposits. For transmittances less than 0.50 these patterns consist of opaque ($D > 3.0$) circular field elements on a transparent ($D = 0$) background. These circular field elements represent developed silver grains or densely clustered clumps thereof. The center distance between adjacent field elements is constant and as those elements increase in size the average transmittance of the sample decreases. The average transmittance (or integrated density) for each sample can be computed since the diameters of the circular opaque spots are known as well as the distance between their centers. This computed value will be referred to as the average *areal* transmittance or integrated areal density. The samples having transmittances less than 0.50 consist of circular transparent field elements on an opaque background. The transparent elements correspond to "holes" in a developed silver deposit of relatively low average transmittance, the opaque background representing the area which is completely filled with developed silver grains. Here, also, the average areal transmittance can be computed from the known dimensions of the configuration, the distance between centers of the adjacent transparent field elements being constant and the same as the distance between the adjacent opaque elements in the samples of average areal transmittance greater than 0.50.

TABLE V. Comparison of syzygetic density differences, $S\Delta D$, with usual density differences, ΔD, for scanning areas of variable size.

Diameter area on sample (μ)	Av. $S\Delta D$	Av. ΔD	Av. $S\Delta D$/Av. ΔD
5.3	0.35	0.25	1.40
10.7	.16	.11	1.45
20.2	.096	.066	1.45
39.9	.045	.032	1.41
79.8	.030	.023	1.30
158.0	.012	.012	1.00

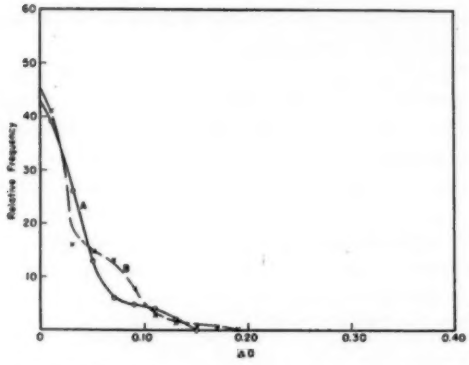


FIG. 23. The relative frequency of occurrence of ΔD measured with a scanning aperture equivalent to 80μ on the sample for two high speed photographic materials of approximately the same sensitivity.

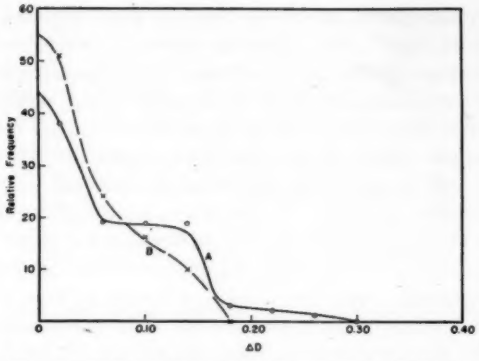


FIG. 24. Relative frequency of occurrence of ΔD, measured with a scanning aperture equivalent to 5.3μ on the sample, for the same pair of photographic materials illustrated in Fig. 23.

The graininess of each sample was determined by the measurement of the blending distance. For the purpose of illustration we have chosen four of these samples, out of a total of eleven, varying in integrated density from 0.002 to 2.5. In the second column of Table VI are given values of average areal transmittance, $\bar{T}$, and in the third those of integrated areal density, ID. The values of relative graininess, J, derived from the measured blending distances are in the fourth column.

By using the standard formula for the spread of illuminance due to diffraction and the following parameters: focal length of the normal eye ($f=17.2$ mm), pupil diameter of 2.5 mm, and distances between the centers of the adjacent opaque areas (or between adjacent transparent areas), it is possible to compute the average distribution of illuminance incident on the retina when the respective samples are at the blending distance. This has been done and the results are shown in Fig. 26. The curves, A, B, C, and D, correspond to the samples thus designated in the first column of Table VI. All of these curves are approximately sinusoidal in shape, the amplitudes decreasing progressively as the average transmittance of the samples decreases. The abscissa scale shows the distance on the retina in microns (μ). Curve B, for Sample B which has an average areal transmittance of 0.50 ($ID=0.3$), is drawn so that x, the distance from crest to crest of the sine wave, is 3μ on the retina. This is equal to twice the value of CD, the distance between centers of adjacent cones in the central region of the fovea. This condition approaches closely to that actually found by making a microdensitometric trace (see Curve B, Fig. 16) on a developed silver deposit, having an average transmittance of approximately 0.50, using a scanning aperture corresponding to a retinal cone projected onto the sample at the blending distance. This condition also agrees fairly well with that derived from the blending distance determined for the conventionalized sample, provided the angular subtense of the sample at the blending distance is of the order of two degrees or more, the surrounding field having a luminance approximately equal to the average luminance of the sample.

Sample C has a blending distance the same as

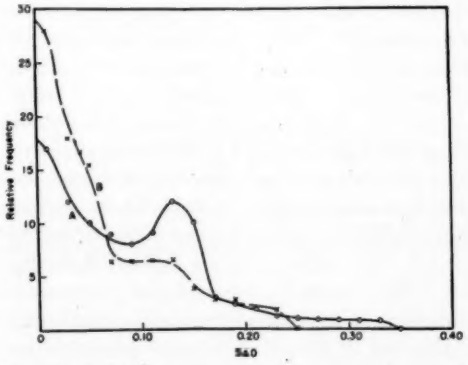


FIG. 25. Relative frequency of occurrence of syzygetic density differences, $S\Delta D$, obtained with a scanning aperture equivalent to 5.3μ on the sample, for the same pair of photographic materials illustrated in Figs. 23 and 24.

that of *B*, hence the distance x from crest to crest of the sine wave is the same as that for sample *B*. Samples *A* and *D* each have a blending distance which is possibly 6 percent of that for Samples *B* and *C*. Hence the crest-to-crest distance x for these samples, as shown in column 5, is 4μ on the retina, this being equivalent to 2.7 *CD*. Curve *D*, which represents the distribution of illuminance on the retina when the total transmittance of the sample is 0.01, if drawn to the same scale as the other curves in this figure, is indistinguishable from a straight line. In the lower right-hand corner of Fig. 26 a section of this curve has been redrawn using an ordinate scale expanded by a factor of 50. In columns 6 and 7 of Table VI are given the maximum and minimum values, in relative terms, of the average illuminance, $\bar{I}$, incident on the retina; and in column 8 are shown the ratios of maximum to minimum average illuminance, $R=\bar{I}_{max}/\bar{I}_{min}$. It will be seen that R is constant, being independent of the average transmittance of the samples, and, therefore, independent of the visual field luminance.

The visual stimuli represented by the four curves in Fig. 26 are all *threshold stimuli*, each one being capable of evoking the perception of *threshold graininess*. Since R is constant it might be concluded that it is the ratio of maximum to minimum illuminance which is the significant characteristic of these stimuli. However, we know that the sensitivity of the retinal receptor is dependent upon the illuminance level to which it is adapted. Since the average illuminance incident on the retina for these four stimuli varies from 90 to 0.01, it does not seem possible that a constant R can be the *only* aspect of these stimuli which imparts to them their threshold character. An inspection of the curves in Fig. 26 indicates that they differ appreciably with respect to amplitude. The values of amplitude, $\Delta\bar{I}=\bar{I}_{max}-\bar{I}_{min}$, are given in the ninth column of Table VI.

TABLE VI. Characteristics of threshold graininess stimuli obtained from conventionalized graininess patterns.

1 Sample	2 $\bar{T}$	3 *ID*	4 Rel. *J*	5 $x(\mu)$	6 $\bar{I}_{max}$	7 $\bar{I}_{min}$	8 R	9 ΔI	10 $\bar{G}_s$	11 Av. G_s
A	0.90	0.045	66	4	94.0	86.0	1.1	8.0	4.0	2.1
B	0.50	0.30	100	3	52.5	47.5	1.1	5.0	3.3	1.5
C	0.10	1.0	100	3	10.5	0.95	1.1	1.0	.67	0.32
D	0.01	2.00	66	4	1.04	0.96	1.1	0.08	.04	0.02

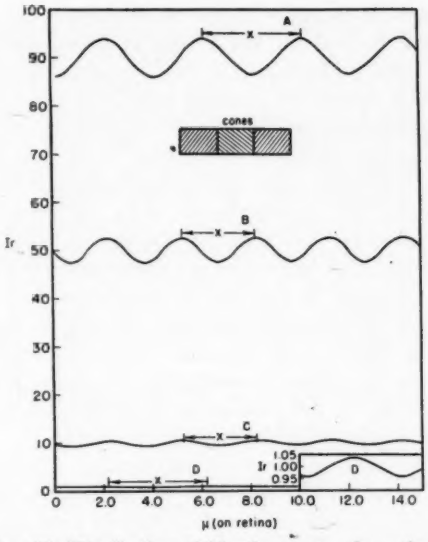


FIG. 26. Distribution of illuminance on the retina for threshold graininess stimuli obtained from conventionalized graininess patterns.

These decrease rapidly as the average transmittance of the sample decreases or as *ID* increases. Also the magnitude of the neural response, elicited when a retinal cone is stimulated by light, is for a given change (amplitude) in the stimulus determined by the suddenness (time rate of change) with which the change occurs. The rate at which the illuminance changes from *point* to *point* on the retina is given by the gradients of the curves in Fig. 26. As an index of this rate we shall use the average gradient, $\bar{G}_s$, although it may be found eventually that the maximum gradient, max. G_s, is a more relevant mode of expressing this factor. However, since the four curves in Fig. 26 are all approximately sinusoidal, the maximum gradient must be proportional to the average gradient. The average gradients of the four curves are shown in column 10. These gradients are spatial gradients expressed in terms of distance, s, measured on the retina and are not equivalent to the time rates at which the illuminance changes on a receptor when that receptor is moving with respect to the illuminance pattern incident on the retina. If we accept the best available data concerning the characteristics of the physiological nystagmic movement of the eyeball, then it appears that the average velocity with which

a central receptor moves with respect to the image formed on the retina is approximately 0.6μ per millisecond. If the receptors were infinitely small, the average time rate of change of illuminance would be given directly by multiplying the numbers in column 10 by 0.6. But this is not the case, for a central receptor has a diameter of approximately 1.5μ (or less but probably not less than 1μ). Moreover, the time rate of change in the average illuminance incident on a receptor is not directly proportional to the spatial rates because the ratio of cone diameter to the crest-to-crest distance, x, is not the same for the four cases shown in Fig. 26. Using a CD value of 1.5, the time rates of change, $\bar{G}_t$, have been computed and are given in column 11.

The average temporal illuminance gradient, $\bar{G}_t$, to which the receptors are subjected when viewing these four conventionalized patterns at the blending distance changes (in relative terms) from 2.1 in the case of Sample A to 0.02 for Sample D. This indicates that when the retinal receptors are adapted to the lowest level of illuminance (0.1 for Sample D) the illuminance gradient required to evoke the perception of threshold graininess is only 1/100 of that required when the receptors are adapted to the highest illuminance (90.0 for sample A). The values of $\bar{G}_t$ are measures of the relative sensitivities of the retinal receptors (when adapted to various luminance levels) to the time rate of change of illuminance incident thereon. The reciprocals of these $\bar{G}_t$ values, when plotted against the average field luminance, determine the *gradient sensitivity* function.

The *gradient* sensitivity of the retinal receptor is, we believe, a new concept in the field of visual sensitometry. Unlike the well-known *contrast* sensitivity and the *acuity* sensitivity, the gradient sensitivity increases as the adapting field luminance decreases. The contrast sensitivity as usually defined is given by the reciprocal of the Fechner fraction, $\Delta B/B$, where ΔB is the just-perceptible luminance difference between the two juxtaposed parts of the test field, and B is the average luminance of the test field. This sensitivity increases rather rapidly for values of B lying between 0.00001 and 1 foot-lambert, and for values greater than 1 foot-lambert remains practically constant up to at least 10,000 foot-lamberts. Acuity sensitivity increases quite rapidly throughout the photopic region as the value of the visual field luminance increases. In spite of these apparent differences in behavior of the three sensitivities in question, contrast and acuity sensitivities are closely related to the gradient sensitivity. But it seems likely that the gradient sensitivity is of more direct fundamental significance from the standpoint of graininess perception than either of the other two types of visual sensitivity. Moreover, it is interesting to note that the values of the *instantaneous luminance threshold* decrease as the luminance of the adapting field decreases, this relationship being of the same general type as that between the gradient values which evoke the threshold perception of graininess and the luminance of the adapting field. The instantaneous luminance threshold has been studied in considerable detail by Nutting,[40] Blanchard,[41] and others. It may be found, when more data on the gradient sensitivity are available, that it is closely related to the instantaneous threshold sensitivity.

From the results obtained with these conventionalized patterns it appears that these threshold stimuli are characterized not only by a constant ratio of maximum to minimum illuminance, but also by a temporal gradient characteristic which depends for its value upon the average luminance of the visual field when the samples are at or very near the blending distance.

We have also computed the distribution of illuminance incident on the retina when the conventionalized configuration represented by Curve B in Fig. 26 is viewed at a distance 33 percent less than the blending distance, Curve A, Fig. 27, and at a distance 33 percent greater than the blending distance, Curve C, Fig. 27. Curve B in Fig. 27 is the same as B in Fig. 26, which represents the spatial illuminance distribution from point to point on the retina when this particular configuration is viewed at the blending distance. We know by direct observation that when the viewing distance is 33 percent less than the blending distance, the graininess in the visual field is *very* perceptible and that when

[40] P. G. Nutting, "1919 report of standards committee on visual sensitometry," J. Opt. Soc. Am. **4**, 55 (1920).

[41] J. Blanchard, "The brightness sensibility of the retina," Phys. Rev. **11**, 81 (1918).

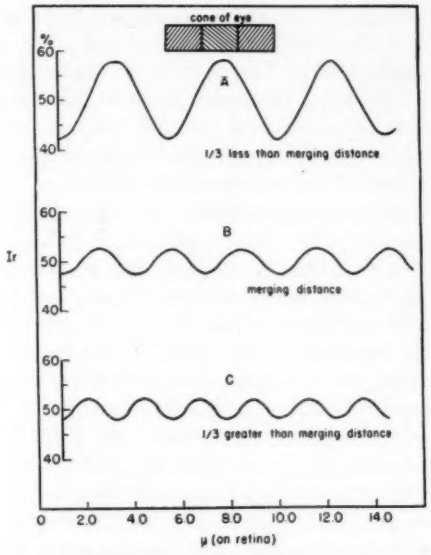


FIG. 27. Distribution of illuminance incident on the retina when the conventionalized sample corresponding to curve *B*, which applies to the merging distance, is viewed at a distance 33 percent less than the merging distance and at a distance 33 percent greater than the merging distance.

the viewing distance is 33 percent greater than the blending distance the graininess has been depressed far below the level of perceptibility. The values of illuminance amplitude, ΔI, of the ratio of maximum to minimum illuminance, R, of the average spatial gradients, $\bar{G}_s$, and of the average temporal gradients, $\bar{G}_t$, incident on the individual receptors, are given in Table VII. The values pertaining to Curve *B* represent the graininess threshold. The values of ΔI, R, $\bar{G}_s$, and $\bar{G}_t$ for Curve *A* are appreciably greater than those for the graininess threshold condition. These values, then, represent the characteristics of a stimulus which evokes perceptible graininess. The value of ΔI for Curve *C* is slightly less than that for *B*, and the same is true for the value of R; $\bar{G}_t$ is also somewhat smaller than for Curve *B*. We know that this visual stimulus is definitely subliminal with respect to graininess. The value of $\bar{G}_s$ for Curve *C* is somewhat greater than that for Curve *B*. This is the spatial gradient incident on the retina and obviously it is not relevant as the characteristic which controls the perceptibility of graininess. It must be remembered that the curves in Fig. 27 represent the illuminance incident from point to point on the retinal surface, while the values of $\bar{G}_t$ are obtained by scanning these curves with an aperture of finite size corresponding to the diameter of a retinal receptor projected on the conventionalized configuration at the three different viewing distances. In the case of Curve *C*, Fig. 27, the size of the retinal receptor relative to the periodicity of the wave form is such that the temporal illuminance gradient, $\bar{G}_t$, is depressed and is in conformity with the observed fact that under this particular condition the graininess is below the level of perceptibility.

It appears now that the time rate of change of the average illuminance, $\bar{G}_t$, incident upon the retinal receptors is the most significant characteristic of the illuminance distribution curves shown in Figs. 26 and 27 with respect to the ability of these stimuli to evoke the perception of threshold graininess. It must be remembered that $\bar{G}_t$ depends for its value not only upon the spatial distribution of illuminance on the retina, but also upon the ratio of the cone diameter, CD, to the periodicity of the illuminance distribution, x. It is entirely possible to maintain the shape of the illuminance distribution with respect to the retina, thus keeping ΔI, R, and $\bar{G}_s$ constant and to obtain widely different values of $\bar{G}_t$ by varying the ratio of the cone diameter to the periodicity. As mentioned previously, in the stimulus represented by Curve *C* in Fig. 27, the average spatial gradient is somewhat greater than in the case of Curve *B*, but this is not sufficient to maintain or increase the visibility of graininess in the case of the *C* stimulus. It is also possible to vary values of ΔI and R without changing the value of $\bar{G}_t$, and, although we have no numerical examples to illustrate these cases, we are convinced that such changes would neither increase nor decrease the visibility of graininess in the visual field, provided the value of $\bar{G}_t$ remains constant. This conclusion is supported by the few data available. It also appears reasonable on

TABLE VII. Characteristics of visual stimuli corresponding to the curves shown in Fig. 27.

Curve	ΔI	R	$\bar{G}_s$	$\bar{G}_t$
A	15.0	1.35	7.1	3.7
B	5.0	1.10	3.3	1.5
C	4.0	1.08	3.6	1.2

theoretical grounds if we assume that the magnitude of the neural response occurring in a visual receptor is determined by the time rate of change of the average illuminance incident thereon. We wish to point out that the curves in Fig. 16 already contain the x to CD ratio and do not represent illuminance gradient from point to point on the retina but show directly the variations in the values of $\bar{G}_t$.

The three curves in Fig. 27 represent a condition which corresponds in a general way to the three curves, A, B, and C, in Fig. 16, which were obtained by scanning an actual developed silver deposit. The two sets of curves cannot be compared quantitatively because they are expressed in terms of different abscissa units, one (Fig. 16) being in terms of the *average log illuminance* incident upon the retinal receptors, and the other (Figs. 26 and 27) in terms of illuminance values incident from *point* to *point* on the retinal surface. The general similarity, however, is striking: in both cases, the shortened viewing distance results in increased amplitudes and gradients, while the decreased viewing distance results in depressed amplitudes and gradients. When we consider the disparity between the methods used in obtaining the two sets of values of $\bar{G}_t$ shown in Tables IV and VII, it is surprising that the relative values of these temporal gradients in one case agree as well as they do with the relative values in the other. This agreement is particularly striking when it is remembered that the values of $\bar{G}_t$ obtained by computation, in Table VII, are based upon the use of certain simplifying assumptions.

We are quite willing to admit that the case represented by the conventionalized configurations, on which our computations represented by Figs. 26 and 27 are based, is an oversimplification of practical conditions, and some of the simplifying assumptions which were made may not be entirely justifiable when we come to consider the problem of actual silver deposits. However, we believe that the results are strongly indicative of the nature of visual stimuli which are responsible for threshold graininess, and, as such, they should serve as useful guides in formulating techniques for the measurement of the objective characteristics of developed silver deposits which will make it possible to obtain the desired correlation with the graininess characteristics of these same samples. It should be remembered that the graininess-density function of a photographic material is determined with constant illuminance incident on samples of variable average transmittance with the consequence that the average luminance of the visual field is dependent upon the average transmittance of the individual samples.

We are inclined to believe that further and thorough investigations will show that the gradient sensitivity of the retinal receptors is a unique function of the average luminance to which the visual system is adapted and that it is independent of the configuration *per se* of the luminance distribution in the visual field. This is equivalent to saying that the gradient sensitivity is the same for all photographic materials. If this proves to be the case, then it begins to be evident that the usual graininess-density function as reported in our previous communications can be broken down into two distinct and independent components.

It should be possible to determine, once and for all, the first of the two components, the *gradient sensitivity versus average field illuminance*. This will involve the application of psychophysical methods (employing a large group of observers and the standardization of the boundary conditions) similar to those used in the establishment of the well-known luminosity function on which international agreement was obtained many years ago. This psychophysical function has now become one of the cornerstones of photometry, colorimetry, and illuminating engineering.

The second component of the total graininess-density function is also a psychophysical relationship It can be determined separately for each photographic material using methods which eliminate the effect of variable gradient sensitivity. This will involve the use of a technique in which the field luminance is held constant, irrespective of the transmittance of the sample. This component must show the relationship between graininess and the configuration of the pattern in the visual field (including the size, shape, and distribution of the developed grains and clumps thereof) plus any existing *variations in the transmittance* of these visual field elements.

It appears probable that the problem of devising a technique for measuring granularity to give direct correlation with this second component will be somewhat simpler than a technique to give direct correlation with the total graininess-density function. There are, therefore, two general approaches to this problem: one, the establishment of an objective method for measuring granularity which will correlate directly with the total graininess-density function, and the other, the establishment of a technique which will give direct correlation with the second component of the total graininess-density function.

The methods of measuring graininess which have been discussed thus far all involve a technique which reduces the grainy appearance of the developed silver deposit to a threshold value, that is, to the point where the grainy structure is just barely perceptible. These techniques all lead to the expression of graininess in terms of the viewing distance at which graininess is just perceptible or in terms of the magnification which produces the same appearance. When results are expressed in terms of these numbers, the question may arise as to whether or not different photographic materials are placed proportionately upon a subjective scale of graininess. Thus, if we have materials to which graininess values of 10, 20, 30, etc., are assigned, the question may be asked, is the graininess difference between the numbers 10 and 20 equal to the graininess difference between the numbers 20 and 30? This, then, raises the question as to the possibility of establishing a graininess scale on which a given difference at one point on the scale is equal to the same difference at another point on the scale. This question presents a new psychophysical problem, which, while difficult, is by no means impossible of solution. It seems likely that eventually it will be necessary to establish a graininess scale on which the unit differences are equal. The only solution of this problem may be to adopt the just noticeable difference with respect to graininess between two samples as the unit of the desired subjective scale. There seems to be no *a priori* method for predicting from objective measurements the exact nature of an *equalized step* graininess scale. Again, we must resort to the psychophysical method and determine by the direct judgment of a sufficiently large group of observers the relation between, let us say, a blending distance-graininess value and the just perceptible or equally perceptible graininess steps.

From a consideration of the relatively few and perhaps inadequate experimental values reported in the previous pages, we are convinced that the most relevant characteristic of a visual stimulus, from the standpoint of its capacity to evoke the perception of graininess, is the time rate of change of illuminance which acts upon the retinal receptors as they move either by conscious effort, or more probably by virtue of the physiological nystagmus relative to the illuminance pattern incident on the retina. If this is the case, then an objective method for measuring granularity should yield information which can be interpreted in terms of these temporal illuminance gradients. For this purpose we believe the syzygetic density or transmittance differences are admirably adapted. Each syzygetic difference can be interpreted as a temporal luminance gradient. The frequency of occurrence of the syzygetic differences, as illustrated in Figs. 17 to 22, inclusive, therefore contain complete information as to the ability of a given silver deposit to generate illuminance gradients on the visual receptors. The problem confronting us now is the accumulation of a large volume of this type of data using a series of adjacent pairs of scanning apertures varying in size and a series of samples differing over the necessary density range for each of a large number of photographic materials. The total generalized graininess function for all of these samples must then be determined by using a threshold method involving either variable viewing distance with fixed magnification, or fixed viewing distance with variable magnification. The latter now appears to us to be more sound than the former. An analysis of these experimental results should make it possible to isolate the region of significant syzygetic differences as illustrated in Fig. 21, and to evaluate the function, Z, in such a way that it will lead to the determination of the ability of a given silver deposit to generate luminance gradients which are capable of evoking the perception of threshold graininess. We venture to predict that the measurement of syzygetic differences will

involve the use of scanning apertures varying in size, or the equivalent variable magnification, for different photographic materials and probably for different densities of the same photographic material. This may appear at first glance to represent a difficult experimental technique to apply for the rapid determination of graininess. However, this is not necessarily true since it seems quite possible to use a pair of adjacent apertures of fixed size on the plane of which can be imaged the photographic deposit in question at variable magnification. With properly designed objective photometric equipment, the magnification can be determined at which the desired value of the Z function is obtained.

It appears possible that the use of a pair of adjacent scanning apertures of fixed and properly chosen size can be used to obtain a series of values of the Z function which will correlate satisfactorily with the second component of the total graininess-density function (or possibly with the total graininess-density function itself) when measurements are confined to a single photographic material or to a group of photographic materials similar to each other with respect to the coarseness of the granular structure of the developed silver deposit. However, it seems very unlikely that a pair of adjacent scanning apertures equal in size can be selected which will yield the desired results for the entire gamut of photographic materials which must be studied.

The data presented here concerning graininess and granularity are fragmentary. They are not sufficiently numerous to warrant any prediction at this time as to how the illuminance amplitudes and gradients may be combined to give results directly significant of graininess. The instrumental means available for the determination of the syzygetic density or illuminance differences and of the rate of change of illuminance on the elements of the retinal mosaic under various conditions of observation are not adequate for the accumulation of a large volume of statistical data which are needed for the solution of this problem. With the existing equipment, the time required for the precise establishment of even one of the frequency curves shown in Figs. 17 to 22, inclusive, is excessive. Steps have been taken, however, to correct this situation and a new instrument, known as a microprojector, has been designed and is nearing completion. The $S\Delta D$ or $S\Delta I$ values can be measured directly for pairs of adjacent, approximately circular areas on the sample. The distance between the centers of the pairs of these areas can be varied from 2μ to 40μ by successive square-root-of-2 steps. The sensitivity is such that $S\Delta D$ values up to 3.0 can be measured even when the center distance between the adjacent apertures is as low as 2μ. The measurements can be made at the rate of 20 or more per minute. Values of ΔD or ΔI may also be measured directly if desired. The instrument is provided with a score board on which the readings are recorded automatically in the form of frequency curves.

The instrument and method which have been used for many years in these Laboratories for measuring graininess, while yielding results of considerable usefulness, leave much to be desired. Since the design and construction of the instrument some twenty-five years ago, we have learned much concerning the mode of functioning of the visual system and relative to the objective characteristics of developed silver deposits. We are convinced that the method should be modified considerably in order to improve the significance and reliability of the measurements. To that end an instrument has been designed differing in a good many respects from the one now in use. It seems probable that the principle of variable magnification rather than that of variable viewing distance is preferable. Binocular observation also is desirable, together with a test field of fixed angular size surrounded by a large, uniformly illuminated area as a means of controlling the general adaptation level of the observer's visual system. Such an instrument has been designed and is nearing completion. In using this instrument we propose to follow closely the conventional procedures used in psychophysical work for the determination of the psychometric function. Such a method requires that the observer who decides as to the perceptibility or otherwise of graininess in the field must have no part in the manipulation of the equipment beyond responding *yes* or *no* to the question, "Can you see graininess?" A large number of such answers made with variable magnifications serves to establish with certainty

the psychometric function, and by using a group of observers we hope to determine graininess values with much greater certainty and perhaps of more direct practical significance than is now possible. With this pair of complementary instruments we hope to accumulate a large volume of data which will be reported in subsequent communications.

In closing, we wish to state again the object of our search: to obtain an objective method of measuring granularity which will give values, either directly or indirectly by the application of a suitable psychophysical function, which are proportional to values of graininess measured throughout the density scale of a given photographic material, and to values of graininess measured on the entire gamut of photographic materials in which we are interested.

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A Method for Making Precise Resolution Measurements*

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A method for making precise resolving power measurements has been developed. This method makes it possible to take into account the ill effects of stray light as well as the common optical aberrations. The method utilizes a device referred to as "The Kinetic Definition Chart Apparatus" (hereafter abbreviated to K.D.C. apparatus). The K.D.C. apparatus provides a means of continuously varying the angular subtense of the markings of a test object without changing the apparent distance between the test object and the optical device under test. Provision is made to vary the contrast and the brightness level of the test object. In order to simulate natural conditions, the test object is surrounded by a region, referred to as "the artificial sky," which may be illuminated to any desired brightness level. The Artificial Sky fixes the brightness level at which the observer's eye is adapted, and provides a means of determining the ill effects on resolution caused by stray light that would normally be present in the optical device under consideration. Fixtures have been made which make it possible to measure the resolution at various field angles for an optical device under test. By means of auxiliary magnification, it is possible to test an optical device at full aperture and practically to eliminate differences among observers by reducing the diameter of the pupil of the observer's eye to less than 1 mm. For comparison purposes, a high grade telescope, referred to as "the standard telescope" is used. The standard telescope is practically free from aberrations and stray light and has practically no loss in light transmission. This telescope provides a means of acquainting an observer with an image of high quality and resolves 4.51 seconds for each inch of aperture. A quantity, referred to as "the K.D.C. efficiency," is introduced to describe the performance and the quality of optical devices. The K.D.C. efficiency may assume two different forms depending upon the end use of the data. In predicting the performance of an optical device, such as the telescope, K.D.C. efficiency is a measure of how much farther a given object is visible using the instrument than with the unaided eye. In the case of quality control of optical devices being mass produced, it is convenient to compare the limiting distance of resolution of the device under test with that of the standard telescope having the same entrance pupil as the instrument under test. The two definitions of K.D.C. efficiency are equivalent if the diameter of the pupil of the eye being used in the observations is less than 1 mm. The method described above has been used to determine the performance and quality of telescopic systems, photographic systems, eyepieces, and microscope objectives, and to measure the resolving power of the human eye under a variety of conditions.

INTRODUCTION

DURING World War II a need arose for an impersonal method of evaluating and controlling the quality of large numbers of optical systems being mass produced. A method of quality control *and for* predicting the field performance of optical devices was developed at the Optical Inspection Laboratory of The Pennsylvania State College. This method is based on

* The work described in this paper was carried out, in part, under Contract OEMsr-1197 between the Office of Scientific Research and Development and The Pennsylvania State College.

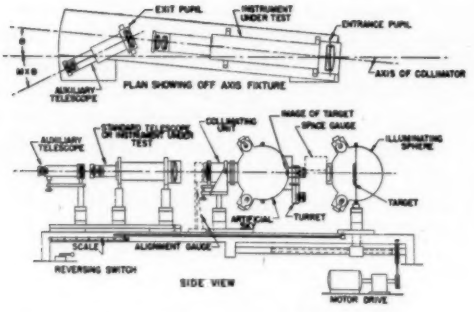

FIG. 1. Schematic diagram of Kinetic Definition Chart apparatus.

the precise measurement of resolving power. The development was made at the request of the Frankford Arsenal and was conducted under the auspices of Section 16.1 of the NDRC.

The method to be described has a number of novel features not usually considered in conventional resolution measurements. These include the use of the following:

1. Test objects in which the angular subtense of the markings can be varied continuously over a wide range.
2. Test objects of different contrasts as viewed against their background.
3. A 30° region surrounding the test object which may be illuminated to any desired level to simulate natural surrounds.
4. A system of auxiliary magnification which stops the pupil of the eye of the observer to such an extent as to render the eye a substantially perfect optical system.
5. A mechanism assembly used to make resolution measurements at various field angles.
6. A method of controlling the over-all angular subtense of the test object as presented to the eye.
7. An optical system which does not require correction factors in the resolution measurements to take into account aberrations and stray light introduced by the testing equipment.
8. A procedure which is rapid, impersonal, and does not require great experience.
9. Versatile assemblies for testing the quality and performance of a variety of optical devices including telescopic systems, photographic systems, eyepieces, microscope objectives and the human eye.

The method of measuring resolution to be described utilizes an apparatus referred to as "The Kinetic Definition Chart Apparatus" which is hereafter abbreviated to the K.D.C. apparatus. The basic principles upon which the K.D.C. apparatus depends are described by Fabry.[1] The K.D.C. apparatus has been used extensively in Army, Navy, and NDRC programs in which thousands of optical instruments were studied and which involved several million resolution measurements. It is the purpose of this paper to describe the K.D.C. method of measuring resolution and to illustrate its application.

THE APPARATUS

The K.D.C. apparatus is shown schematically and photographically in Figs. 1 and 2. It consists of a test object, an optical reduction unit, a collimating unit, a standard telescope, an auxiliary telescope, an Artificial Sky, two gauges, a mechanism whereby the spacing between the angular subtense of the test object markings can be varied continuously, an off-axis fixture, and an observing microscope.

THE TEST OBJECT

The test object illustrated here was introduced by Foucault.[2] This object consists of alternate white and gray bands of equal width and which are of the form shown in Fig. 3. In general it has been found desirable to use test objects in

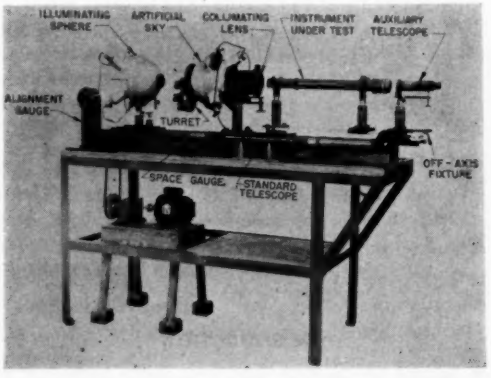

FIG. 2. The Kinetic Definition Chart apparatus.

[1] C. Fabry, Proc. Phys. Soc. London **48**, 747 (1935).
[2] Leon Foucault, Ann. de L'Observ. de Paris, 197, *t.v.* 1858.

which the markings are oriented in four directions. The relatively coarse numbers shown in Fig. 3 are for purposes of identifying the test object and for focusing. The K.D.C. test objects can be made having any given spacing and contrast between the white and gray bands. These are calibrated photoelectrically for the number of bands per unit length and for contrast. The test object is illuminated by means of a sphere as shown in Fig. 1.

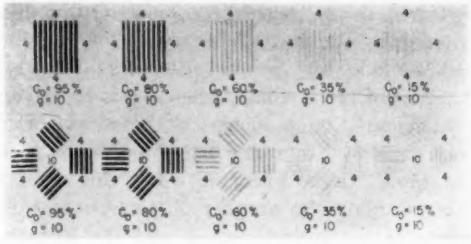

FIG. 3. Kinetic definition chart test objects.

$C_0 = \frac{B_0 - b_0}{B_0}$ $\frac{\text{reflectivity of white band} - \text{reflectivity of dark band}}{\text{reflectivity of white band}}$.

C_0 = inherent target contrast. g = No. of white bands/inch.

THE OPTICAL REDUCTION UNIT

The apparent size of the test object is reduced optically by means of any one of a set of lenses mounted in a turret shown in Figs. 1 and 2. These lenses are referred to as the optical reduction unit. The apparatus in its present form utilizes microscope objectives having focal lengths of 4, 8, 16, and 32 mm, and Plössel type eyepieces having focal lengths of 0.784 and 1.918 inches. The optical reduction unit is required in order to obtain practical limiting distances of resolution. As a result of this feature, the entire length of the K.D.C. apparatus is approximately 6 feet. Early models were as long as 120 feet.

THE COLLIMATING UNIT

The collimating unit is simply a high grade telescopic objective having a focal length of 15.75 inches and a clear aperture of 3 inches. The purpose of the collimating unit is to make it possible to test a given optical device for any reasonable apparent distance. The distance between the collimating lens and the optical reduction unit can be varied by means of a threaded joint.

THE STANDARD TELESCOPE

The standard telescope is an astronomical instrument consisting of highly corrected optical parts and provided with stray light stops. The objective has a clear aperture of 2.5 inches and an effective focal length of 15.75 inches. Provision is made for varying the aperture of the standard telescope by means of a set of stops so that its entrance pupil can be made equal to that of the optical device under test.

The purpose of the standard telescope is to establish the minimum angle of resolution for an optical system of a given aperture and to acquaint the observer with the appearance of an image formed by a high grade optical system. Using a test object of high contrast, the standard telescope resolves an angle of 4.51 seconds at an aperture of 1 inch. At the limit of resolution, the contrast between the light and dark bands imaged on the retina of the observer's eye is approximately 1 percent. For well illuminated objects the numerical values of the angle for an aperture of 1 inch and for the limiting contrast of the retina of the human eye are in agreement with the values reported by Dawes[3] and by Blackwell.[4]

THE AUXILIARY TELESCOPE

The auxiliary telescope is an astronomical telescope having an entrance pupil of 1.250 inches. The focal length of the objective is 5.00 inches. A set of eyepieces are used having focal lengths ranging from $\frac{1}{8}$ of an inch to 3 inches. The auxiliary telescope is normally used in series with the instrument under test and hence its entrance pupil is equal to the exit pupil of the instrument under test.

In the quality control of optical devices, the auxiliary telescope has three important functions. The first of these is to make it possible to test the system under consideration at full aperture by reducing the exit pupil of the system under test to less than that of the natural pupil of the eye. The second is to reduce the portion of the pupil of the eye being used in the measurement to such an extent (1 mm or less) that the eye

[3] Rev. W. R. Dawes, Me. Roy. Astronom. Soc. **35**, 154 (1867).

[4] H. Richard Blackwell, J. Opt. Soc. Am. **36**, 624 (1946).

performs as a "perfect" optical device. This practically eliminates differences in measurements made by various observers. The third function of the auxiliary telescope is to produce a sufficiently large image on the retina of the observer's eye to make an observation "comfortably." It is probable that this comfort depends upon the number of *photosensitive* elements of the retina excited and the length of the boundary of the image of the test object on the retina. This factor is discussed by Fry.[5] In this connection, it should be noted that the auxiliary magnification cannot provide a means of exceeding the limit of resolution of the eye because of the inverse relation between the size of the exit pupil at the eye and magnification and the limitations imposed by diffraction theory.

THE ARTIFICIAL SKY

In order to simulate outdoor conditions, the image of the test object formed by the optical reduction unit is surrounded by an illuminated region. This region is referred to as "The artificial sky" and subtends an angle of approximately 30° measured from the center of the collimating objective. The artificial sky is constructed such that it forms an adjustable field stop for the optical reduction unit. This adjustment is accomplished by means of a series of spherical segments having various apertures and mounted in a turret as shown in Fig. 1.

The artificial sky has two functions. The first is to produce a known brightness level to which the observer's eye is adapted. The second function provides a means of taking into account the stray light normally introduced into an optical system by natural backgrounds when the optical device is used in performing its intended task. It is the stray light in optical devices that is primarily responsible for reducing the contrast of the image on the retina and consequently the range in which objects are visible. The ill effects of contrast reduction have been described by Hardy,[6] Duntley,[7,8] and Blackwell.[4]

[5] Glenn A. Fry, "The Relation of the Length of a Border to Its Visibility," presented at the 31st meeting of the Optical Society of America, October 1946.
[6] A. C. Hardy, J. Opt. Soc. Am. **36**, 283 (1946).
[7] S. Q. Duntley, J. Opt. Soc. Am. **36**, 359 (1946).
[8] S. Q. Duntley, "Contrast Reduction by the Atmosphere Along Inclined Paths of Sight," presented at the 31st meeting of the Optical Society of America, October 1946.

THE VARIABLE SPACING UNIT

The mechanism for controlling the angular subtense of the target markings consists of a motorized unit by which it is possible for the observer to vary the distance between the test object and the optical reduction unit continuously. This variation changes the size of the image formed by the optical reduction unit and consequently the angular subtense of the target markings. The distance between the optical reduction unit and the test object is indicated on a scale graduated in tenths of an inch. The optical parts are arranged such that the scale reading is directly proportional to the minification of the image of the test object formed by the optical reduction unit.

By virtue of the continuous feature of varying the distance between the test object and the optical reduction unit, precise resolution measurements can be made. This procedure eliminates the use of the discreet intervals of conventional resolution objects.

THE GAUGES

In order to facilitate the adjustment of the K.D.C. apparatus two special gauges are used. These are shown in Figs. 1 and 2.

The first of these gauges, referred to as the space gauge, is used to set the distance between the target and the optical reduction unit such that the scale reads linearly with magnification. This gauge is simply a metallic block five inches long. Appropriate scale readings have been determined for each of the optical reduction units when the gauge is placed between the face plate of the test object illuminator and the cell of the optical reduction unit.

The second gauge, referred to as the alignment gauge, provides a means of aligning the test object, the optical reduction unit, and the collimating unit. This gauge is essentially a two dimensional square and height gauge which can be placed on the ways of the casting of the K.D.C. apparatus as indicated schematically in Fig. 1.

THE OFF-AXIS FIXTURE

The K.D.C. apparatus is provided with a mechanical assembly for mounting the optical

system under test and making resolution measurements at various field angles. This fixture, referred to as "the off-axis fixture," is designed such that the optical device under test can be aligned with the optical axis of the collimating unit and can be rotated about its own entrance pupil. The off-axis fixture also provides a means of rotating the auxiliary telescope or observing microscope about the exit pupil of an optical device under test.

The purpose of the off-axis fixture is to provide a convenient method for making resolution measurements at various field angles.

THE OBSERVING MICROSCOPE

In measuring the resolution of photographic and telescopic objectives, eyepieces, and microscopic objectives, it is necessary to use an observing microscope. This microscope is of the type used on conventional lens benches.

THE PROCEDURE

The procedure for making resolution measurements using the K.D.C. apparatus depends upon the optical device under test and the end use of the results. Because of the wide variety of optical devices which may be tested for resolution, the present discussion is limited to describing procedures for testing complete telescopic systems, photographic and telescopic objectives, and eyepieces. It might be mentioned that microscope objectives, prisms, and mirrors have been tested for resolution using the K.D.C. apparatus.

DEFINITIONS OF K.D.C. EFFICIENCY

In presenting resolution measurements using the K.D.C. apparatus, it has been found useful to introduce a numerical rating of an optical instrument under consideration. This numerical rating is referred to as the K.D.C. efficiency. The K.D.C. efficiency is conveniently used in two different forms depending upon whether the resolution measurements are to be used for predicting the performance or for determining the quality of an optical device under consideration. For example, in predicting the performance of telescopic systems, it is desirable to know how much farther a given object can be seen with the instrument under consideration than with the unaided eye. This point of view gave rise to the definition of the K.D.C. efficiency given by Eq. (1).

$$\text{K.D.C. efficiency} \equiv \frac{\alpha_e}{\alpha_i M_i} \times 100, \qquad (1)$$

where:

α_i = minimum angle resolvable using instrument under consideration.

α_e = minimum angle resolvable using the eye alone.

M_i = magnification of the instrument under consideration.

Equation (1) may be rewritten in the form shown by Eq. (2) in which the scale readings of the K.D.C. apparatus are substituted for the angles in Eq. (1).

$$\text{K.D.C. efficiency} \equiv \frac{X_i}{M_i X_e} \times 100, \qquad (2)$$

where:

X_i = K.D.C. scale reading at limit of resolution for the instrument under test and the eye.

X_e = scale reading at limit of resolution for the unaided eye.

In determining maximum resolution of an optical system, auxiliary magnification is generally used. Since the ultimate resolving power of an optical instrument depends upon only the aperture of the system and the wave-length of light used, the auxiliary magnification does not increase the resolution beyond the limit set by diffraction. The auxiliary magnification does, however, stop the pupil of the eye thereby making it more perfect optically. The auxiliary magnification should be such that not more than 1 mm diameter of the pupil of the eye is used during the measurement. Under these circumstances, the K.D.C. efficiency defined by Eq. (2) is equivalent to the definition presented in Eq. (3).

$$\text{K.D.C. efficiency} \equiv \frac{X_i}{S} \times 100, \qquad (3)$$

where:

X_i = K.D.C. scale reading at the limit of resolution for the instrument under test using the auxiliary telescope.

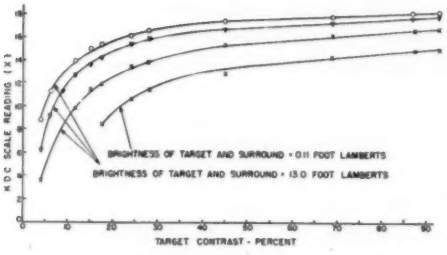
K.D.C. SCALE READING (X)
BRIGHTNESS OF TARGET AND SURROUND = 0.11 FOOT LAMBERTS
BRIGHTNESS OF TARGET AND SURROUND = 13.0 FOOT LAMBERTS
TARGET CONTRAST - PERCENT

FIG. 4. K.D.C. scale readings (X) *vs.* target contrast for a standard telescope with and without artificial sky.

aperture = 1.00 inch
18.2 scale units = 4.51 seconds resolved
4 mm optical reduction unit
15.75 inches—focal length collimating lens
25 lines per inch

Legend
o—clean optics, without sky
▽—thumb print on objective, without sky
x—thumb print on objective, with sky

S = K.D.C. scale reading at the limit of resolution for the standard telescope having the same entrance as the instrument under test and using sufficient auxiliary magnification to eliminate the imperfections which may be introduced by the observer's eye.

It will be noted that when one uses Eq. (3), he is simply comparing an instrument under test with one known to be of high optical quality and to be practically free from stray light. It should be noted, however, that since the standard telescope has a constant specific coefficient of resolution (defined to be the product of the aperture in inches and the minimum resolvable angle in seconds) of 4.51 inch seconds, the K.D.C. efficiency may be converted into angular units.

THE PROCEDURE FOR TESTING TELESCOPIC SYSTEMS

The procedure for measuring the resolution for telescopic systems may be divided into two parts. The first of these is the measurement of resolution for test objects of various contrasts at some given field angle, and the second is the measurement of resolution at various field angles with a test object of fixed contrast.

In measuring the resolution of the telescopic system at various contrasts, measurements are usually made on the optical axis only. The procedure consists simply of noting the K.D.C. scale readings for the limit of resolution of a set of test objects of various contrasts. The results of such measurements are usually plotted in the form shown in Fig. 4. Generally, it is desirable to make resolution measurements with and without the Artificial Sky in operation and at least two different brightness levels. In order to illustrate the importance of surface defects and cleanliness a thumb print was placed on the center of the objective of the standard telescope to provide a source of stray light as this instrument is practically free of such defects. In the measurements presented, uniform surround was used and the target was at the same apparent brightness as the surround. This is not generally the condition found in nature and the ill effects caused by stray light presented here are conservative since the surround is frequently at a higher brightness level than an object being observed. Figure 5 is presented to illustrate the relation of K.D.C. efficiency to the scale readings. It is to be noted that the K.D.C. efficiency is 100 percent by definition for the standard telescope with clean optics.

In measuring the resolution of telescopic sys-

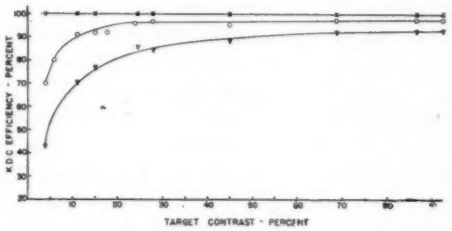

FIG. 5. K.D.C. efficiency *vs.* target contrast for standard telescope with thumb print on the front of the objective.

aperture = 1.00 inch

Legend
x—standard telescope—without thumb print
o—without sky
▽—with sky

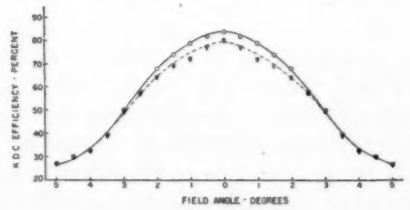

FIG. 6. K.D.C. efficiency *vs.* field angle for 3× telescope with and without artificial sky.

aperture = 0.828 inch
100 percent K.D.C. efficiency = 4.51 seconds × aperture in inches

ems at various field angles, the procedure consists of simply noting the K.D.C. scale readings for the limit of resolution at each field angle for a test object of fixed contrast. In addition to determining off-axis K.D.C. efficiencies, measurements were made with and without the Artificial Sky in operation at each field angle. The K.D.C. data are presented in Fig. 6. These data illustrate the relative importance of the reduction in resolution of stray light and the off-axis aberrations.

If the purpose of the test is to determine the maximum resolution of an optical device, the auxiliary magnification should be selected such as to yield the greatest K.D.C. scale reading. It has been found that for an instrument of poor quality, or for a target of low contrast, the amount of auxiliary magnification that will yield optimum results may be as much as half that value used for a high grade instrument or for a target of high contrast. A general rule may be followed for high contrast targets of using a total magnification of 50× for the instrument under test and the auxiliary telescope combined for each inch of aperture of the instrument under test.

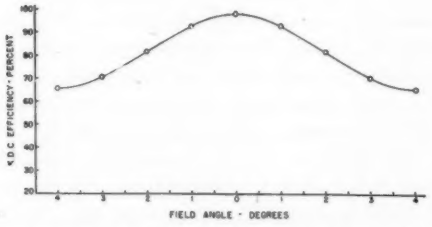

FIG. 7. K.D.C. efficiency *vs.* field angle for 5× telescope without auxiliary magnification.

100 percent K.D.C. efficiency = 12.8 seconds exit pupil larger than pupil of the eye

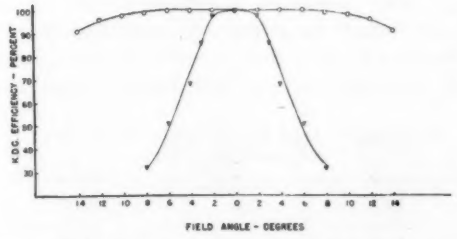

FIG. 8. K.D.C. efficiency *vs.* field angle for objectives.

100 percent K.D.C. efficiency = 4.51 seconds × aperture in inches
o—camera objective, 8×10 tessar I_a, 11¼″ E.F.L., *f*:4.5 stopped to 0.792 in.
▽—telescope objective, 11.45″ E.F.L., aperture stopped to 0.730 in.

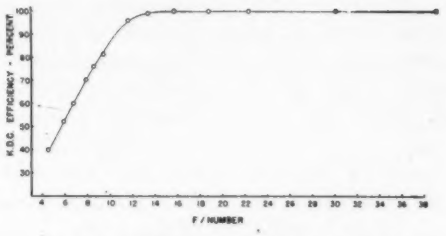

FIG. 9. Axial K.D.C. efficiency *vs.* *f*:number for tessar objective.

8×10 inch tessar I_a
E.F.L. = 11.75 inches

In case the resolution measurements are to be used in predicting the performance of the optical device under test, no auxiliary magnification is used. In this case the limit of resolution is determined for the unaided eye and then for the instrument under test. The K.D.C. efficiency is then computed by Eq. (2). The numerical values of the K.D.C. apparatus scale readings obtained for the resolution of an optical instrument without auxiliary magnification may be taken to be proportional to the range at which target is visible. Typical data for a 5× telescope without auxiliary magnification are presented in Fig. 7.

THE PROCEDURE FOR TESTING TELESCOPIC AND PHOTOGRAPHIC OBJECTIVES

The procedure for testing telescopic and photographic objectives is essentially the same as for testing telescopic systems. The principle difference lies in the fact that instead of an auxiliary telescope being used, a microscope is used to observe the image formed by the objective under test. It is convenient to make the measurements at various field angles by rotating the lens about its rear nodal point in the same manner as the conventional lens bench is used. Typical resolution data at various field angles for telescopic and photographic objectives are shown on Fig. 8. In the case of photographic objectives, it is of interest to measure the K.D.C. efficiency at various *f*:numbers. Typical data of this type are shown in Fig. 9.

THE PROCEDURE FOR TESTING EYEPIECES

The procedure for testing eyepieces is essentially the same as that for testing objectives

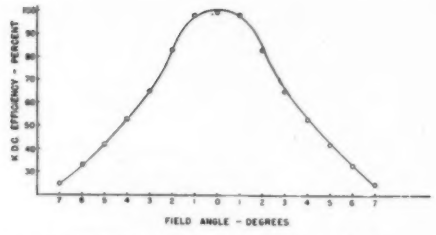

FIG. 10. K.D.C. efficiency *vs.* field angle for eyepiece (eyepiece reversed and tested as an objective, using a stop equivalent to the exit pupil for the instrument in which the eyepiece is used). Stop diameter 0.275 in.

100 percent K.D.C. efficiency = 4.51 seconds × aperture in inches

except that the eyepiece is tested as an objective and with light from the K.D.C. apparatus entering the eyepiece through a stop equivalent to the exit pupil in size and position. Typical data obtained for an eyepiece are presented in Fig. 10.

THE DATA

Typical data obtained using the K.D.C. apparatus are shown in Figs. 4–10 and in Table I. These data illustrate the results of resolution and K.D.C. efficiency measurements under a variety of conditions for telescopic systems, telescopic and photographic objectives, eyepieces, and microscope objectives. Since the data presented here are merely to illustrate the K.D.C. method, an analysis is not made.

POSSIBLE ERRORS

There are two principal sources of error in making resolution measurements using the K.D.C. apparatus. The first of these is in focusing on the image and the second is the failure to expose the image of the object to the eye long enough. The error in focusing has been reduced by focusing on numerical figures rather than on the resolution markings of the test object. The exposure time required is at least 10 seconds for low contrast objects or for poor quality images. This error is evidently related to unsteadiness of the observer's head as the exposure time is reduced by using a chin rest. If the above errors are minimized, the resolution measurements made using the K.D.C. apparatus do not exceed two percent in the case of optical devices of high quality.

DISCUSSION AND CONCLUSIONS

The conclusions reached concerning the K.D.C. method of measuring resolving power of optical devices may be summarized in the following brief statements:

1. The K.D.C. method provides a simple procedure for making precise resolution measurements at different contrasts and brightness levels of a variety of test objects.

2. The K.D.C. method may be used for quality control purposes and to measure the resolving power of a variety of optical devices including telescopes, photographic objectives, microscope objectives, eyepieces, and the human eye.

3. The K.D.C. method takes into account ill effects on resolution caused by stray light in optical devices as well as the optical aberrations.

4. The K.D.C. method provides a means of objectively evaluating the design and of predicting the performance of optical devices.

MISCELLANEOUS COMMENTS

In connection with the development and use of the K.D.C. apparatus, the following miscellaneous comments are made:

1. An early model of the K.D.C. apparatus consisted of a motorized test object mounted on a 50-foot track. This preliminary model was free from the objection of using optical parts which might influence the resolution measurements. Extensive measurements clearly show that the absolute values of resolution, as well as the K.D.C. efficiency determined by the early and later models, differ less than the probable experimental error of two percent.

2. During the War an experiment was conducted to test the impersonalness of the K.D.C. apparatus. A number of tank sights of different quality were inspected by approximately 25 different company and government officials using four different units of the K.D.C. apparatus.

TABLE I. Axial K.D.C. values for 32-mm microscope objectives.

Objective No. 1	100%
Objective No. 2	93%
Objective No. 3	93%
Objective No. 4	95%
Objective No. 5	99%
Objective No. 6	96%

The results of the experiment indicated no appreciable difference in the resolution measurements for the different K.D.C. units and for the different observers.

3. The case of one observer who had the lens of one of his eyes removed should be cited. This observer was able to make K.D.C. measurements as well as normal observers when a final exit pupil of less than 1 mm was used. This indicates that the lens of the eye contributes very little to the K.D.C. observations if the final exit pupil is sufficiently small.

4. The K.D.C. apparatus has been used extensively for measuring the resolving power of the human eye for a variety of artificial pupil stops for monocular vision at various brightness levels and for test objects of various types and contrasts.

5. Preliminary results indicate that by using test objects of various shapes, the K.D.C. apparatus may be used to predict the range at which objects would be detected as well as the range at which objects may be recognized.

6. The K.D.C. method has been found useful in studying the loss in resolution caused by known amounts of specific aberrations. In the case of the primary aberrations in objectives, it has been found that the loss in resolving power is from 4 to 10 percent per Rayleigh limit depending upon whether the objectives are over or under corrected. For example, the under corrected spherical aberration of the photographic objective referred to in Fig. 9, was 1.5, 3.0, and 5.0 Rayleigh limits at f:10, f:8, and f:6, respectively. It will be noted that in this case the decrease in K.D.C. efficiency is close to 10 percent per Rayleigh limit.

7. In using Foucault test objects the criterion for the limit of resolution was taken to be that angle at which all the directions of target markings could no longer be determined. This criterion leads to reproducible results for optical devices of all qualities provided the markings are oriented in meridians 45° apart and the optical device under test is focused on numerals of the type shown in Fig. 3.

8. It has been found that the ill effects of aberrations in optical devices become increasingly great as the contrast of the test object is decreased.

ACKNOWLEDGMENT

The authors wish to acknowledge the interest and assistance received from the personnel at the Frankford Arsenal. Over fifty individuals at the Frankford Arsenal contributed to the development of the K.D.C. apparatus. The Frankford Arsenal has ruled that these individuals should not be mentioned specifically.

The authors are indebted to Dr. F. C. Whitmore, Dean of the School of Chemistry and Physics at The Pennsylvania State College, for constant interest and encouragement during the development of the K.D.C. apparatus.

Acknowledgment is made of the interest and suggestions received from the following individuals: Dr. Theodore Dunham, Jr., Chief of 16.1 NDRC, Dr. F. E. Wright, Section member of 16.1 NDRC, Colonel F. S. Brackett, Medical Research Laboratory, Fort Knox, Dr. I. C. Gardner, Bureau of Standards, Commander S. S. Ballard, Bureau of Ordnance, Navy Dept., Dr. D. M. Packer, Naval Research Laboratory, Lt. Commander N. H. Pulling, Bureau of Ordnance, Navy Dept., Dr. Richard Tousey, Naval Research Laboratory, and Mr. C. W. Brouse, Dr. W. R. Ham, Mr. E. M. Kelly, Dr. D. H. Rank, Mr. Joseph Vrabel, and Dr. H. L. Yeagley, all of The Pennsylvania State College.

The Production and Calibration of Foucault Test Objects of Various Contrasts*

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(Received December 14, 1946)

A method is described for the production and calibration of resolution test objects of a type similar to those introduced by Foucault. These test objects consist of alternate white and gray bands of equal width. The targets are photographic copies of impressions made from inked ruled zinc plates. In order to compensate for the lateral diffusion of the white bands in the photographic processes, the zinc plates are ruled to produce black bands twenty percent wider than the white bands. In the copying process each photograph consists of alternate exposures to the zinc plate impression and to a white card, such that the total exposure for each photograph is constant. A series of such photographic copies is simultaneously printed on one sheet of photographic paper. These prints constitute a set of test objects of various contrasts in which the white and gray bands are practically equal in width. The test objects are calibrated by means of a photoelectric mechanism which records the reflectivity of the white and gray bands and indicates the number of white bands per inch. Using the method outlined above, test objects have been produced and calibrated in which the number of lines varied up to 120 per inch and which had contrasts as low as 1 percent.

INTRODUCTION

THE performance of optical instruments is often expressed in terms of resolving power. In determining the resolving power of an instrument experimentally, it is common practice to use a high contrast object of a type introduced by Foucault.[1] This type of test object consists of a set of alternate white and black bands of equal width. The form of these objects has been generalized to include various contrasts between the bands. Modified forms of the test objects are shown in Figs. 1 and 2. The different contrasts are needed to relate the field performance of optical instruments to resolution measurements.

The production of Foucault test objects which are uniform in contrast and in spacing is difficult if one requires a white background and equal widths of the white and gray bands. It is the purpose of this paper to describe a simple method that has satisfactorily been used to produce and calibrate a large number of Foucault test objects for the Army, the Navy, and the Office of Scientific Research and Development.

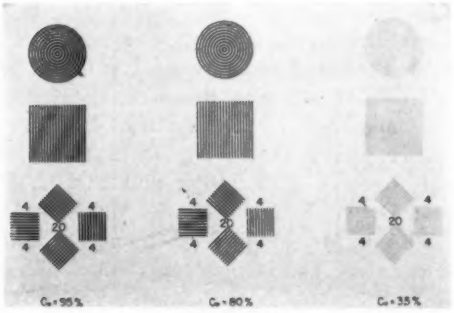

FIG. 1. Modified Foucault resolution targets.

$$Co=\frac{Bo-bo}{Bo}=\frac{\text{Reflectivity of white band}-\text{reflectivity of gray band}}{\text{reflectivity of white band}}\times 100.$$

Co = Inherent target contrast.

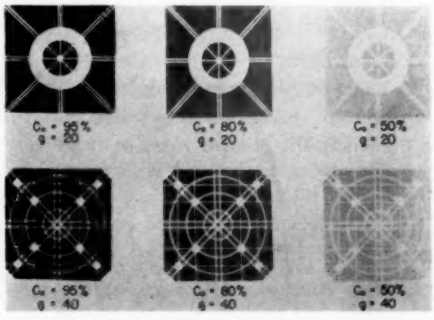

FIG. 2. Penn State resolution test objects.

$$Co=\frac{Bo-bo}{Bo}=\frac{\text{Reflectivity of white band}-\text{reflectivity of gray band}}{\text{reflectivity of white band}}\times 100.$$

Co = Inherent target contrast.
g = No. of white bands per inch.

* The work described in this paper was carried out, in part, under Contract OEMsr-1197 between the Office of Scientific Research and Development and The Pennsylvania State College.

[1] L. Foucault, Ann. de L'Observatoire de Paris 197, *l.v.* (1858).

THE PRODUCTION OF FOUCAULT TEST OBJECTS

The method used to produce the Foucault test objects illustrated in this paper involves three processes. These include the machining of a zinc plate, the making of ink impressions of the zinc plate, and the photographing of the impressions.

The machining operation consists of the ruling of square grooves in a zinc plate. The machining is done on a shaper for straight bands, and on a lathe for circular bands. The grooves are twenty percent less in width than the metal between them for the objects shown in Figs. 1 and 2.

The ink impressions from the machined zinc plate are made by using a conventional printing press.

A set of photographs is made of the ink impression. The negatives made may be on separate films if desired. The first negative is exposed such that when it is contact printed, with full development, the black and white bands of the print are equal in width. Each remaining negative is exposed for less time to the ink impression and is re-exposed to a white card. The re-exposure time is chosen such that the sum of the exposure times for each negative equals that of the first negative. A typical set of exposure times is shown in Table I. By means of such a technique, there is obtained a series of photographic negatives of the same maximum density but on which the contrast of the images varies. These may be printed either simultaneously or individually yielding targets of the form shown in Figs. 1 and 2.

It might be noted that other methods of controlling the contrast have been successfully used. These include the use of rotating sectors, rotating cylinders, and the introduction of stray light into the camera during the photographing of the ink impression by means of a clear diagonal glass plate.

TABLE I. Exposure times and contrasts for a set of photographs of ink impressions of the ruled zinc plate.

Time of exposure			Contrasts on		
			Ansco commercial film	Eastman Kodabromide	
To ink impression	To white matte card	Total		F-1 paper	F-4 paper
67 sec.	61 sec.	128 sec.	75%	65%	51%
52 sec.	76 sec.	128 sec.	65%	52%	33%
43 sec.	85 sec.	128 sec.	54%	42%	25%
31 sec.	97 sec.	128 sec.	42%	29%	14%
17 sec.	111 sec.	128 sec.	20%	12%	5%

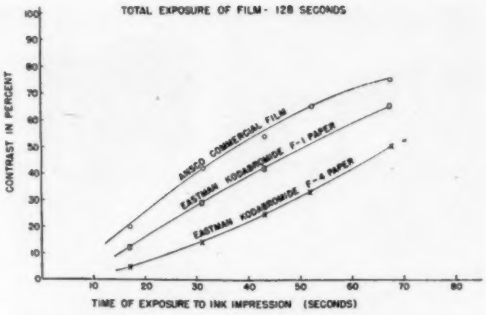
TOTAL EXPOSURE OF FILM - 128 SECONDS
CONTRAST IN PERCENT
ANSCO COMMERCIAL FILM
EASTMAN KODABROMIDE F-1 PAPER
EASTMAN KODABROMIDE F-4 PAPER
TIME OF EXPOSURE TO INK IMPRESSION (SECONDS)

FIG. 3. Image contrast *versus* time of exposure to ink impression.

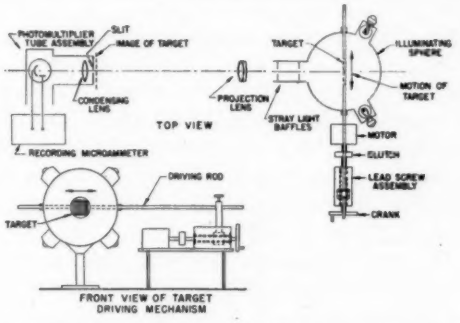

FIG. 4. Schematic diagram of calibration apparatus for Foucault type resolution targets.

Table I and Fig. 3 show the contrasts of the images of the negatives and the prints for a typical set of exposures. A graph, as shown in Fig. 3, is used to determine the exposure time necessary to produce a test object of any desired contrast. Satisfactory test objects have been produced having contrasts as low as one percent.

The number of bands per unit length is varied by photographic reduction. Ruled zinc plates have been made as large as 18 inches square, producing test objects having up to 120 white bands per inch.

THE CALIBRATION

The Foucault test objects are calibrated both for contrast and for the number of white bands per unit length. The calibration utilizes the apparatus shown schematically in Fig. 4 and photographically in Fig. 5.

The apparatus includes an illuminating sphere and a holder for the test object which is motarized to produce a translation as shown schematically

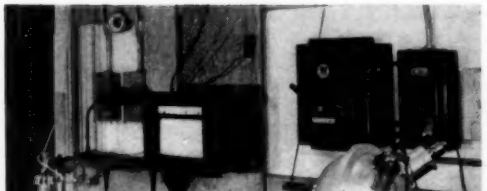

FIG. 5. Apparatus used to calibrate resolution targets.

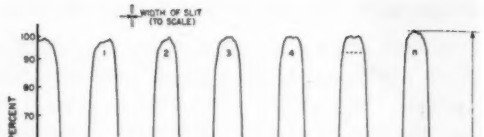

FIG. 6. Typical calibration curve for a Foucault test object

$$\text{Inherent contrast } (Co) = \frac{Bo - bo}{Bo} \times 1000.$$

$$\text{No. of white bands per inch } (g) = n/x.$$

in Fig. 4. An image of the test object is projected by means of a lens on to the slit which forms the entrance pupil to a photo-multiplier tube. The photoelectric response is indicated by means of a recording microammeter, giving rise to a record of the type shown in Fig. 6.

The procedures for analyzing the curve to obtain the contrast and spacing of the test object are indicated on Fig. 6. If the spacing or contrast varies across the test object, it is possible to obtain values for specific bands.

THE DISCUSSIONS AND CONCLUSIONS

1. The method described for producing and calibrating Foucault test objects has been found satisfactory. The error in calibrating high contrast test objects is less than 1 percent and for low contrasts (less than 3 percent) it may be as much as twenty percent.

2. Using the method described above, a large number of targets has been produced, calibrated, and used by the Army, the Navy, and the NDRC. These targets were used primarily in the testing of optical instruments and in the measuring of the resolving power of the human eye.

3. It has been found that the use of the Foucault test objects is equivalent to using point sources in the measurement of resolving power. The principal advantage in using the Foucault test object over point sources lies in the precise knowledge of the conditions under which the resolution measurement is made.

ACKNOWLEDGMENT

Acknowledgment is made of the suggestions received as to the form of the test objects from Dr. Theodore Dunham, Jr., and Dr. F. E. Wright of Section 16.1 of the NDRC, and Dr. I. C. Gardner of the Bureau of Standards.

Acknowledgment is made of the assistance received from Mr. C. W. Brouse in connection with the design and construction of the equipment described.

Acknowledgment is made of the assistance and interest of Mr. R. H. Breon of the Penn State Camera Shop in connection with the photographic procedures described.

The Stiles and Crawford Effect in Polarized Light

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(Received December 6, 1946)

IN a very recent paper Bárány[1] describes a most interesting experiment with the retina in polarized light intended as a test of a suggestion made by Wright and Nelson[2] in 1936 to account for the Stiles and Crawford effect. The suggestion and brief discussion by Wright and Nelson point out that light may be totally reflected within a retinal element (in the manner originally described by Brücke) and that if the angle of incidence is too great light may escape from the element into the interspace and become inactive. Actually such escape into the medium between the rods and cones will not result in lowered effectiveness of the light and less sensation of brightness, unless some concentration of the energy density of radiation within the element (retinal cone) has first taken place. Without such concentration a cone will on the average gain as much light from adjacent elements as it loses from itself, with no net change in the visual effectiveness of the light. It appears necessary that such concentration into the outer segment of the cone occur in the manner recently described by the writer,[3] and that the principal absorption resulting in cone vision take place in this outer segment, if a quantitative theory of the Stiles and Crawford effect is to follow.*

Although not described by Bárány as such, his recent experiment is equally applicable in this case also. If his interpretations are correct his results appear to invalidate the present theory as well as the proposal by Wright and Nelson which it was intended to test. For this reason it is of interest to examine his interpretation in some detail.

When the angle of incidence is less than the critical angle and light escapes from a medium of higher to one of lower refractive index, the amount escaping will depend upon the plane of polarization as will also the amount reflected within the medium of higher index. The quantities reflected and escaping through a simple plane interface can be calculated very easily from the Frésnel equations, but as Bárány points out, the calculations and numerical prediction for the more complicated shapes of the retinal rods and cones might prove a difficult matter. Nevertheless, with this concept it seemed reasonable to expect that if a portion of the retina were illuminated by a single oblique pencil of rays from the margin of the pupil, and if the light entering the eye were plane polarized, the visual response would vary with the orientation of the plane of polarization. Bárány has conducted this experiment with what appear to be all the necessary precautions, and observed no fluctuation in the visual sensation even when the polarizer was rotated several times per second.

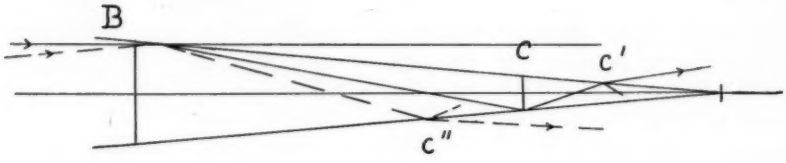
This, he states, argues against reflection within a cone as an explanation of the Stiles and Crawford effect, although pointing out that the evidence is not conclusive "since it is not known how large the anticipated and not demonstrated effect would be if the theory were correct."

In Fig. 1 is shown a simplified geometrical representation of the posterior end of a human retinal cone cell. The portion from A to B represents cone body; that from B to C, the so-called cone ellipsoid; and that from C to D, the outer segment. A cone cell at or just beyond the margin of the fovea might be represented by an included angle for the truncated portion of about 8 degrees, while a cell farther from the fovea, by an angle of about 12 degrees. In Fig. 2 is shown on a larger scale the region from B to C.

[1] E. Bárány, Acta Ophthalmol. **24**, 1 (1496).
[2] W. D. Wright and J. H. Nelson, Proc. Phys. Soc. **48**, 401 (1936).
[3] * After this manuscript had been completed a letter was received from Professor Wright stating that "I had in mind exactly the process you describe but didn't amplify the suggestion." The suggestion and entire discussion in the paper by Wright and Nelson is contained in a single sentence "Alternatively the light may be totally reflected inside the walls of the cone, and when the angle of incidence becomes too great the light escapes outside the cell and is inactive." It was not apparent from this that the necessity for light concentration into the outer segments of the cone cells had been considered at all, nor the fact that the useful absorption of the light must occur in these outer segments to explain the effect. Nevertheless, it is a pleasure to acknowledge Professor Wright's letter and to note that although he did not describe it in his publication, he had a similar explanation in mind.

FIG. 1.

FIG. 2.



It is evident that light entering parallel to the axis of the cone will be internally reflected at B, and will reach the surface of the cone again at C. If the ratio of refractive index of cone to surrounding medium is sufficiently great, the ray will then be totally reflected at C and can thus enter the outer segment. If, however, an oblique ray as shown by the dotted line enters the conical region at B, it will be reflected and will strike the surface of the cone again at C'' at a point nearer the base, and at a smaller angle of incidence which may be less than the critical angle. It is this ray which will escape and to which the experiment of Bárány will apply if the entering ray is polarized.

The calculation of Bárány's "anticipated effect" is a laborious task even for the simplified geometrical shape of a simple cone frustum. Fortunately, however, it is a comparatively simple matter to determine the effect experimentally by the use of a model. It is only necessary to provide a transparent frustum of a cone of appropriate angle, and to surround this cone with a medium of lower refractive index. Collimated polarized light can then be projected obliquely on the base of the cone, and the quantity emerging from the smaller end of the frustum measured as a function of the plane of polarization of the incident light.

If the model cone is made of material which is strongly birefringent, any fluctuation in the quantity of radiation emergent from the cone with direction of the plane of polarization might be masked by the effects of the birefringence. The use of most transparent plastics or ordinary glass rod is thus ruled out. While a transparent cone could be made of good quality optical glass ground and polished on all surfaces, it was found simpler to make the cone of transparent liquid in a cell of transparent plastic of lower index than the liquid. If this cell is properly designed any birefringence in the plastic will be unimportant, since light once entering the plastic will be trapped and not emerge from the cone model.

The construction of such a model cell is shown in Fig. 3 together with the schematic optical arrangement for measuring the transmitted light. The cell body is formed by a rod of polished lucite (polymerized methyl methacrylate), C, which is bored out to form a conical cavity of the appropriate angle, the walls of which are polished. Black metal diaphragms limit the entrant and emergent beams, and the cell is closed with plate glass windows held in place by threaded end rings. The cell is filled with a liquid of refractive index higher than the lucite, and is so mounted that it may be illuminated at any desired angle, θ, off axis, by a light source S consisting of a ribbon filament tungsten lamp and collimating lens. A polarizer P mounted in front of the light source provides for rotating the plane of polarization of the entering light. The emergent light is received by a barrier layer photo-cell at B connected to an appropriate galvanometer.

Some of the rays emerging from the cone will make a large angle with the axis, and the reflection losses at the emergent window might vary with the orientation of the plane of polarization. Furthermore, the barrier layer photo-cell is covered with a plane glass window which will also vary in reflection with the plane of polarization. To eliminate these effects a flashed opal glass disk O, is optically cemented to the emergent window. Thus the photo-cell receives diffuse illumination irrespective of whether the light

enters the cone parallel or oblique to the axis. Moreover, the flashed opal glass to a large extent depolarizes the emergent radiation, and the combined effect is to render photo-cell indications dependent only upon the total light flux emerging, and independent of entrance angle θ or of the orientation of the plane of polarization of the entering light.

If the angle of incidence, θ, of the entering light is appreciable, then the reflection loss at the entrance window of the conical cell will vary with the plane of polarization. For example, if the window has a refractive index of 1.545 and the angle of incidence, θ, is 16 degrees, this will result in 1 percent more light entering the conical cell when the electric vector lies in the plane of incidence than when it is perpendicular to this plane. This is desirable in the test model because of the similar effect of the cornea of the eye on marginal rays as discussed below.

For the purpose of considering polarization effects the anterior surface of the cornea may be approximated by a sphere of radius 7.8 millimeters. Referring to Fig. 4 a ray entering parallel to the optic axis but at a height h above it will be incident upon the corneal surface at the angle θ, where $\sin\theta$ equals h/R. For $R=7.8$ millimeters and $h=4$ millimeters (the position of the aperture as used by Bárány) the angle of incidence θ will be approximately 30 degrees. Although the substantia propria of the cornea has a refractive index of approximately 1.37, the transition from air through the tear layer and corneal epithelium has the effect of reducing the equivalent index for reflection purposes to approximately 1.35. For such an index and an angle of incidence of 30 degrees the difference in the quantity of light entering the cornea for plane of polarization in and perpendicular to the plane of incidence is approximately 2 percent. This is to be compared with the 1 percent difference with plane of polarization at the entrance window for the test cell in the example cited above.

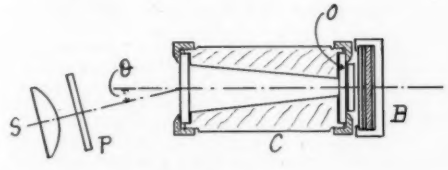


FIG. 3.

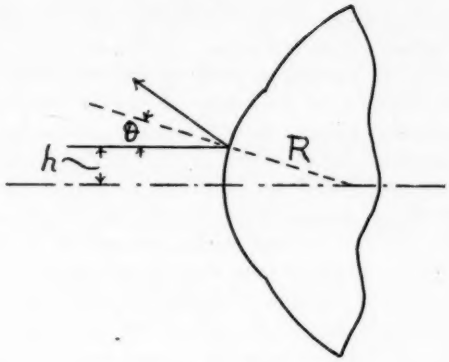


FIG. 4.

Two model cells were made with included angle of 8 degrees and 12 degrees, respectively. In each case the dimensions were made such that the truncated conical portion terminated at the point C as defined by the rays shown in Fig. 2. For the 8-degree cone the ratio of refractive index of liquid to cell wall (i.e., surrounding medium) was made 1.026 while for the 12-degree cone this ratio was made 1.047, the index ratio in each case being chosen so that total reflection just occurred at the point C of Fig. 2. Thus for light incident upon the cell parallel to the axis, all rays entering the cell were just transmitted through the small end of the cone, while for light entering at a small angle θ to the axis some of the limiting rays escaped into the surrounding transparent plastic and were lost. By means of a suitable filter, measurements were confined largely to the yellow and orange portion of the visible spectrum so that dispersion differences between liquid and plastic cell wall were minimized.

Variation in photo-cell current with change in the plane of polarization was investigated for several angles of incidence (external) from 0 to 20 degrees, corresponding to internal angles of incidence up to 12 degrees. The sensitivity of the indicating instrument was sufficient to detect changes of a few tenths of one percent. When the plane of polarization was rotated the deflection was found to be constant within one percent for all angles of incidence investigated for

both the 8-degree and 12-degree cone models. In other words Bárány's experiment repeated with these models in place of the eye yielded, with rotation of the plane of polarization, no fluctuation greater than 1 percent in the light transmitted into what, in the model, would correspond to the outer segment of a retinal cone cell. Since the smallest fluctuation which an observer could detect under the conditions of Bárány's experiment amounts to several percent, the present experiment appears to demonstrate clearly that the magnitude of the effect anticipated by Bárány is in fact much smaller than would be detectable to a human observer even under the most favorable circumstances.

In view of the foregoing it seems quite definite that the experimental results of Bárány are in complete agreement with predictions from the theory recently proposed by the writer to account for the Stiles and Crawford effect. This does not mean, however, that the theory is necessarily correct. However, it does add another piece of experimental evidence, and so at least increases one's confidence in the reasonableness of the theory. It may be of value, therefore, to consider its consequences in the light absorption by the photo receptor substance of the cone cells.

Let us assume the cross section of the outer segment of a cone cell to be one-tenth the area of a cross section of the cone body, and consider a portion of the retina with a high cone population in which the cone bodies are pressed so closely together as to leave no interspace between them. In this case the light flux concentration into the outer segments will be in the ratio of ten to one, so that the outer segments are, in effect, illuminated by light of intensity ten times as great as that incident upon the retina. If the absorbing pigment in the outer segments is at a concentration c, then the light energy absorbed in these outer segments will be the same as that which would be absorbed by a retina consisting of a homogeneous layer of pigment of depth equal to the length of the outer segments of the cones, and at an average pigment concentration equal to c. However, since in this case the volume of the outer segments is only one-tenth that of the volume of such a homogeneous layer, it is evident that only one-tenth the total quantity of pigment is required for a given energy absorption than would be required with a homogeneous pigment layer.

If one goes further and assumes that the quantum efficiency at visual threshold is of the same order of magnitude for absorbed light in cone vision as for rod vision, then the quantity of cone pigment required is indeed small. Since the dark adaptation threshold for cone vision is approximately three orders of magnitude higher than the dark adaptation level for rod vision, only 10^{-4} as much cone pigment need exist in the retina as the quantity of rhodopsin found in an equal area of rod populated retina when fully dark adapted, the additional power of ten resulting from the assumed light flux concentration into the cone outer segments. In view of this the difficulties to be encountered in isolating the cone pigment may be even greater than previously expected, and it may be that only by indirect approaches can the properties of this pigment be determined. In any case if the present theory can be confirmed it will have the effect of localizing the seat of light absorption and cone vision in the outer segments of the cone cells, and thus of localizing the first step in the transformation of light energy into visual response of a light adapted eye.

The absence of a Stiles and Crawford effect in rod vision requires on this theory that there be no substantial light flux concentration into the portion of the rod cells containing rhodopsin. Possible reasons for this have already been mentioned.[3] Experimental evidence is not yet sufficient to permit evaluating these.

[3] B. O'Brien, J. Opt. Soc. Am. **36**, 506–509 (1946).

Spectrochemical Analysis by the Copper Spark Method*

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A system of spectrochemical analysis is described in which 0.1 ml of a hydrochloric acid solution of the sample is evaporated on the ends of a pair of flat-topped copper electrodes which are then excited in a spark. Absolute sensitivities for different elements ranging from 10^{-10} to 10^{-6} gram have been obtained. The lower limits measurable by visual comparison with standard plates are given for 64 elements. The visual method gives results accurate to a factor of two over a range from the lower limit up to about 10 micrograms. The accuracy can be increased, with some loss in sensitivity, by densitometry with an internal standard, an average deviation from the mean of 3 to 8 percent being obtained depending on whether a spectroscopic buffer is used. In addition to high sensitivity the method has the advantage that a great many elements can be determined in one exposure, so that almost any sample which can be obtained in hydrochloric acid can be analyzed for impurities without special calibration. The method was developed for the analysis of plutonium samples and is the result of considerable cooperative effort. The original stimulus was due to Professor G. K. Rollefson of the University of California at Berkeley, whose work was then considerably expanded at Chicago by the authors. Valuable contributions were also made by Dr. Raymond G. Russell, Dr. Wayne W. Marshall, Miss Irene Corvin, and the many analysts who used the method during its evolution. No special mention of the plutonium application is made in the present paper.

INTRODUCTION

THIS paper describes a system of spectrochemical analysis in which a drop of solution is evaporated on flat-topped copper electrodes which are then excited in a spark. There is nothing very novel in the principle and no claim for originality is made. Copper electrodes have been used by a number of analysts, although usually in a d.c. arc to avoid interference from cyanogen bands, but they have never become popular. This is probably because of the difficulty of working with electrodes of low melting point, because the electrodes are less easily machined than graphite electrodes, and because the reducing power of the arc is poorer than with the graphite arc leading to lower sensitivity for substances introduced as salts or oxides. A few references to spark excitation of samples evaporated on copper electrodes can be found in the literature (for example, Gerlach and Riedl[1] were able to show by this means that a 0.44 mg sample of radium contained less than one part per million of barium) but no one apparently had developed the method systematically as described below in regard to sensitivity and completeness of coverage.

The primary objective was the development of a method for the determination of impurities in plutonium. ". . . the plutonium purity requirements are very high indeed,"[2] so that high concentrational sensitivity was essential. Moreover, high absolute sensitivity was required because of restrictions on the sample size imposed by the scarcity of plutonium until the large scale plant at Hanford became operative and by the health hazard due to the associated alpha activity. In addition it was desired to obtain the maximum number of elements with a single exposure, although some impurities were naturally of more importance than others. Definite figures for the concentrations were required, even for those elements which were not detected in a given sample and which were therefore expressed as upper limits. On the other hand high accuracy

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[1] W. Gerlach and E. Riedl, Zeits. f. anorg. Chemie **221**, 103–8 (1934).

[2] H. D. Smyth, *Atomic Energy for Military Purposes* (Princeton University Press, 1945), p. 137.

was not important, a factor of two being usually considered sufficient.

The copper spark method, with modifications, proved to be well suited to the problem. However, the emphasis in the present paper is not on plutonium analysis, partly to avoid delay in the release of this information and partly to indicate one of the most important features of the method, namely, its universality.

PROCEDURE

To serve as an orientation for the following sections which discuss characteristics of the method in some detail, a general description of the procedure will first be presented.

The electrodes are formed from 1.25″ lengths of 0.25″ diameter commercial hard copper rod by machining the ends flat and removing the surface of the sides for a distance of about 0.5″ back from the ends. This operation is conveniently performed on a lathe fitted with a lever-type collet chuck, using a tool appropriately ground so that turning and facing can be accomplished without reorienting the tool. A slight chamfering of the edges is desirable but not essential. No other cleaning, except for a preliminary nitric acid wash, is required, and further attempts at cleaning after machining are in fact detrimental. The machined electrodes are stored in holes drilled in plastic blocks and covered with sawed-off test tubes which seat in annular wells.

The sample, in the form of 0.1 ml of one percent hydrochloric acid solution, is deposited in equal portions on the ends of two electrodes and evaporated. The tendency of the solution to run over the sides is lessened by first dipping the electrodes in a petroleum ether solution of stopcock grease (approximately one gram Apiezon "*N*" per liter). The evaporation equipment consists of coils of ten turns of 20-gauge Nichrome wire wound to support the electrodes and mounted on Bakelite panels. The coils are connected in series and supplied by a variable autotransformer delivering about two volts per coil, the voltage being adjusted so that evaporation is completed before the solution begins to boil.

The exposure conditions are, in general, not critical. Any convenient spark source and spectrograph can be used, the exposure time being chosen by means of moving plates so that it is terminated when most of the sample has been consumed. Fast plates are usually required for maximum sensitivity. The optimum conditions depend somewhat on whether the plates are to be densitometered or examined by visual comparison with plates of known solutions.

SENSITIVITY

All of the variables in the procedure which could be made accessible were investigated for their effect on sensitivity, and will be taken up in order.

Commercial hard copper was chosen for the electrodes after comparison with a number of other likely materials on the basis of simplicity of spectrum, purity, sparking characteristics, ease of machining, and cheapness. Metals having a complex spectrum such as platinum and iron were excluded because of the interference to be encountered in analyzing for a great many elements. Aluminum did not behave well in the spark in that it was practically impossible to find discharge conditions and electrode shapes such that the discharge adequately covered the surface on which the sample was deposited. In addition even the purest aluminum which could be obtained on special request to the Aluminum Company of America (type 7A) contained entirely too much of the more common impurities to be considered. Zinc was another metal which behaved poorly in the spark, the difficulty in this case being caused by oxide formation. Gold and silver were satisfactory but offered no advantage over copper. The special case of graphite is discussed below.

Commercial hard copper rods from a number of sources were tested and all were found to be of adequate purity for the method. Early attempts at cleaning the electrodes by acid washing led to high blanks, varying with the source of the copper, for such common elements as calcium and magnesium. This phenomenon was finally ascribed to adsorption from the acid after machined electrodes were found to have barely detectable amounts of these elements. Zero blanks were obtained in this way for specially pure copper sold by the American Smelting and Refining Company (although the blanks for the acid washed electrodes were also high), but the special copper offers no practical advantage be-

cause the blanks of the purest reagents which could be prepared were always higher for these elements than the traces shown by ordinary copper.

The machined surface of the electrodes should be very smooth to prevent local concentration of the discharge. Various shapes and dimensions have been tried. The surface density of sample deposit can be increased by reducing the diameter, with a slight increase in line to background ratio, but the increased sensitivity is insufficient to warrant the greater evaporation difficulty. End shapes other than flat lead to non-uniform sparking, except possibly for Rogowski electrodes.[3] Completely uniform sparking is not desirable for evaporated deposits, since these tend to concentrate near the edges as the drop dries, and flat-topped 0.25″ electrodes with a slight rounding of the edges with a file or even left sharp are preferred. For the uniform deposits obtained by electrodeposition, a flat-topped 0.125″ counter electrode of copper is recommended.

The sensitivity does not depend critically on the electrical characteristics of the spark source. Increase in the power leads to an increase in the initial intensity but increases the rate at which the sample is consumed and also increases the background intensity. Consequently exposures terminated at about the same sample consumption show a sensitivity almost independent of power. With a conventional controlled spark source and a given capacitance across the secondary of the transformer there is a broad optimum value for the inductance in the oscillating circuit. With very low inductance the spark stays outside the gap and only stray discharge excites the sample. This fact plus the greater air excitation with higher background leads to poor sensitivity. With very high inductance the arc-like discharge consumes more of the electrode relative to the sample and also the total intensity is low. The optimum gap is two millimeters. Suitable circuit constants for a commerical power supply are as follows: Baird Power Source Panel (modified), input to transformer primary with 660-watt heating coil in series, 4.5 amperes at 130 volts; output, 25,000 volts; capacitance, 0.005 microfarads; inductance, 0.036 millihenry; synchronous interrupter with ultraviolet irradiation.

The A.R.L.-Dietert Multisource Unit was found to offer no significant advantage in sensitivity, since best results in general were obtained with an oscillatory discharge with circuit constants in the ratio 5 microfarads to 100 millihenries. The reproducibility, however, is somewhat better than with conventional sources, and the flexibility is convenient in selecting the optimum exposure.

Moving plates show that the sample intensity decreases roughly exponentially except for an initial flash of high intensity and very short duration. The flash probably corresponds to mechanical dislodgement of larger deposit particles but its integrated intensity is only a small fraction of the total. The rate of decrease increases with power and inversely with inductance as indicated above. It is apparent that the highest line to background ratio occurs at the beginning of the exposure, so that the more the exposure is prolonged for a given set of conditions the poorer will be the sensitivity. There is also no advantage in long exposures from the standpoint of precision, since little sample light is being contributed at the end, in contrast to the usual situation in the analysis of metal electrodes in which conditions are adjusted to give as nearly constant intensity as possible. The "half life" for the Baird source specified above is 10–15 seconds.

The background obtained consists of lines of copper and air superimposed on a continuum which is more or less uniform throughout the ultraviolet, falling off toward the red. The line spectrum is relatively intense but is comparatively simple and unimportant for background considerations. Other things being equal, therefore, high dispersion is desirable in order to decrease the intensity of the continuum relative to the lines, since the ultimate sensitivity limit is reached when the line can no longer be distinguished above the background intensity. On the other hand, the above data show that the amount of useful light is limited, so that it is impractical to go to very high dispersion because the speed of the spectrograph becomes very low. It has been found that the three standard makes

[3] J. D. Cobine, *Gaseous Conductors* (McGraw-Hill Book Company, Inc., New York, 1941), p. 177.

of modern commercial grating spectrographs (Baird, Jarrell-Ash, and A.R.L.-Dietert, having first-order dispersions of 5, 5, and 7 A/mm, respectively) are suitable when used with fast emulsions.

Narrowing the slit width is another factor which reduces the intensity of the continuum. However, the spectrographs referred to are of relatively low aperture, and the line intensity is consequently also reduced with narrow slits by diffraction, so that the line to background ratio is almost unchanged over a practical slit width range. A width of 50 microns has been adopted as a compromise between intensity and resolution.

Fast emulsions are used in order to minimize the exposure times. Very fast emulsions have poor storage qualities and poor reproducibility from batch to batch, making them undesirable for quantitative work based on comparison with standard plates containing known concentrations. Experience in the ultraviolet has accordingly been limited to Eastman Spectroscopic Plates, Types I–0 and 103–0. The former has only about half the speed of the latter at low densities but with a longer straight line portion of the characteristic curve and with greater contrast, so that the curves eventually cross.[4] Both of these plates are unfortunately slower than would be desired for use with grating spectrographs and it is necessary to expose longer than the optimum time in order to bring up the background. It is evident that small differences in line intensity above background intensity produce greater differences in density when the background density is on the straight line portion of the characteristic than lower down, so that from the standpoint of the emulsion the optimum exposure for greatest sensitivity produces a background just off the toe. It has been found that this advantage outweighs the reduction in the integrated line to background ratio arising from the longer exposure. Furthermore, background uniformity with wave-length was the reason for the choice of grating over prism spectrographs for this method. On the basis of these considerations the I–0 plates have been used for visual comparison, while the lower grain of the 103–0 plates makes them more desirable for densitometric work.

A number of arrangements of spectrograph irradiation were investigated, including various combinations of cylindrical lenses. It was found best to focus the source on the grating with a short focal length spherical lens. The line intensity is uniform across the spark gap for the a.c. spark.

Various attempts were made to increase the sensitivity by producing a more adherent sample deposit on the electrode in order to minimize mechanical loss in the spark. The addition of substances which dried in the form of a glass, such as phosphoric acid and some organic bases, was tried with indifferent success. Electrodeposition was found to increase the intensity by a factor of about three, as well as providing a means of concentrating trace amounts of certain metals in the presence of other substances which would yield a bulky deposit and so prevent concentration by evaporation.

The results of the above effort are summarized in Table I, in which the limits of detection for various elements are given in milligammas (10^{-9} gram). This unit was found to be of a convenient order of magnitude and has the further advantage that absolute amounts of impurities can be converted easily to parts per million parts of sample by multiplying by the reciprocal of the sample weight in milligrams. For comparison with representative sensitivities using the d.c. graphite arc and a large quartz spectrograph, the results of Cholak and Story[5] are also given.

Figure 1 presents the same data in the form of a periodic table. It is apparent that the limiting sensitivity is in general a periodic function of atomic number. Most of the decrease in sensitivity in going down a given column of the table is caused by the increase in atomic weight. Some irregularities are evident but in most cases are because of the fact that it was not feasible to use the most sensitive line, as discussed below. High sensitivity for strontium, for example, was never required, so that a less sensitive line lying in a region of greater general usefulness was used. In the case of the common elements such as sodium, calcium, and magnesium, the lower

[4] *Photographic Plates for Use in Spectroscopy and Astronomy* (Eastman Kodak Company), fifth edition.

[5] J. Cholak and R. V. Story, J. Opt. Soc. Am. **31**, 730–738 (1941).

TABLE I. Sensitivities.

Element	Copper spark Grating Order	Copper spark Wavelength	Copper spark Sensitivity ($m\gamma$)	D.c. arc** Wavelength	D.c. arc** Sensitivity ($m\gamma$)
Ag				3280.7	10
Al	1	3944.0	10	3082.2	20
As	1	2860.5	500	2780.2	1000
Au	1	2675.9	20		
*B	2	2497.7	10		
Ba	1	3891.8	100		
	1	6142.8	10		
Be	1	3130.4	0.2		
Bi	1	3067.7	20	3067.7	20
Ca	1	3933.7	10		
Cb	1	3163.4	20		
Cd	1	2748.6	200	3261.1	400
Ce	1	4012.4	100		
	2	4186.6	50		
Co	1	2663.5	50		
Cr	1	2843.2	5	2986.0	200
Cs	1	8521.1	50		
Cu				3274.0	20
Dy	2	4000.5	50		
Er	2	3906.3	50		
Eu	1	3972.0	5		
	2	3972.0	2		
*F	1	6856.1	10		
Fe	1	3820.4	100	3020.7	20
	2	4383.5	50		
Ga	1	2943.6	100		
Gd	1	3768.4	50		
	2	3768.4	10		
Hf	1	3877.5	200		
	2	4048.0	50		
Hg	1	2536.5	500	2536.5	200
Ho	1	3891.0	100		
	2	3891.0	20		
In	1	3256.1	100		
Ir	1	2833.2	500		
K	1	7699.0	10		
La	1	3949.1	5		
	2	3949.1	5		
Li	1	6707.8	0.2		
Lu	1	3397.1	200		
Mg	1	2852.1	1	2852.1	10
Mn	1	2949.2	2	2801.1	20
Mo	1	2816.1	5		
Na	1	5895.9	10	3302.0	2000
Nd	1	4012.3	50		
	2	4012.3	20		
Ni	1	2416.1	10	3050.8	400
Np	2	4164.5	200		
P	1	2554.9	2000	2535.6	5000
Pa	1	2744.0	500		
	2	4217.2	200		
Pb	1	2833.1	50	2833.1	20
	2	4057.8	5		
Pd	1	3609.5	50		
Pr	1	4100.8	50		
	2	4225.3	20		
Pt	1	3064.7	2		
Pu	2	3709.1	200		
Ra	2	3814.4	10		
Rb	1	7800.2	20		
Re	1	4360.5	200		
Sb	1	2790.4	500	2598.1	400
Sc	1	4247.0	1		
	2	4247.0	0.5		
Si	1	2881.6	20	2881.6	200
	2	2881.6	10		
Sm	1	4280.0	50		
	2	4280.0	20		
Sn				2840.0	20
Sr	1	3464.6	50		
Ta	1	2685.5	100		
Te	1	2383.2	50		
Th	1	4019.1	50		
	2	4019.1	20		
Ti	1	3761.3	10		
Tl				2767.9	20
Tm	2	3795.8	5		
U	1	3932.0	500		
	2	3932.0	100		
V	1	3093.1	5		
W	1	4008.7	50		
Y	1	3710.3	10		
	2	3710.3	1		
Yb	1	3694.2	10		
Zn	1	3345.0	200	3302.6	2000
Zr	1	3392.0	50		
	2	4149.2	10		

* On CaO deposit.
** See reference 5.

limit of sensitivity has never been reached because of contamination.

It is submitted that the copper spark method in general offers higher absolute sensitivity with greater reproducibility and more complete coverage using one set of conditions. The low sensitivity usually quoted for spark sources refers to impurities in the metallic electrodes and is ascribed to the low consumption of material. In the present case the low electrode consumption is believed to be the chief factor in emphasizing the volatilization of a surface layer of impurities. The same consideration is important in the comparison of copper and graphite as supporting electrodes, since graphite is attacked by the spark to a much greater extent. Graphite is also less satisfactory because of its porosity. Its superiority over copper when an arc is used is ascribed to its higher melting point which prevents the sample becoming inaccessible by incorporation in a molten bead, and to its greater reducing power.

QUANTITATIVE ANALYSIS BY VISUAL COMPARISON

One of the most valuable characteristics of the copper spark method is the absolute reproducibility compared with other high sensitivity

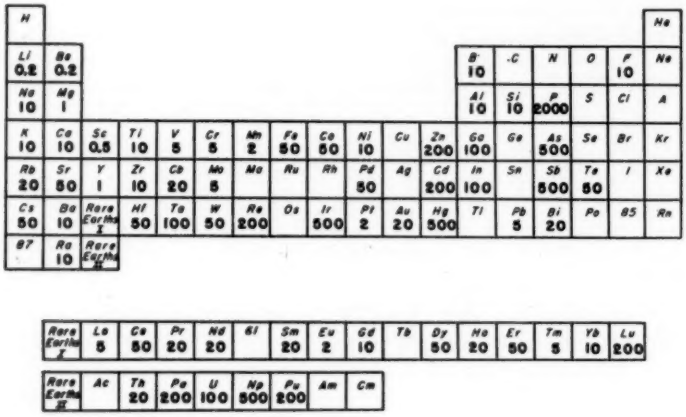

FIG. 1. Periodic table of copper spark sensitivities. Units in milligammas (10^{-9} gram).

sources, subject to conditions enumerated below. This was taken advantage of by preparing a number of standard plates containing one millimeter high exposures of known amounts of the elements decreasing in the relationship 10, 5, 2, 1 and covering the range 10,000 to 0.01 mγ. By mounting these plates with cover plates an effective system was devised, using the dual projector of the A.R.L.-Dietert Comparator-Densitometer, for comparison with plates containing the sample exposures. Interpolation between standards was sometimes justified, especially if the increased accuracy was sufficiently important to make it worth while exposing known solutions as external standards on the same plate with the samples for correction purposes. Even without external standards it was found that with proper control of humidity and photographic processing the error seldom exceeded a factor of two. Relatively large amounts were judged by the widths of the lines which increased noticeably with intensity for dense lines with the plates used. By this means figures could be quoted for the amounts of all the metallic elements in a sample with a single exposure, the great majority, of course, being reported as "less than."

Since the wave-length range accessible with commercial spectrographs with a single exposure does not cover all the sensitive lines of interest, some compromise in sensitivity is necessary for limited samples. This restriction is more serious with the Baird spectrograph than with the Jarrell-Ash, since the latter uses twice the length of plate with the same dispersion. A procedure adopted for the early plutonium samples was the simultaneous use of two spectrographs set for different ranges using the same source. A less satisfactory but more convenient method was the simultaneous photography of first-order red and second-order ultraviolet spectra on the same plate, this procedure resulting in less complete coverage for the ultraviolet due to the greater dispersion. For most applications less sensitive lines are good enough or else the sample can be divided into aliquots for exposures in different wave-length regions.

For the preparation of the standard plates stock solutions of the individual elements were made up in one percent hydrochloric acid at a concentration of one milligram per milliliter. Most of these stock solutions were found to be stable for many months but if one is not the fact is quite visible at this concentration. Successive dilutions were then made with Pyrex mixing cylinders or calibrated quartz test tubes to yield the appropriate concentrations for the standard plates. Each plate contained mixtures of three or four elements at the same concentration, plus an internal standard if desired. In order to be certain of proper wave-length identification for elements with rather complex spectra and doubtful purity, such as the rare earths, weak exposures of the individual elements were also made for comparison, this method proving more reliable than wave-length measurement on the standard plates and comparison with tables.

Investigation of intensity interferences showed

that relatively large amounts of one element suppressed the intensities of trace amounts of others. Elements in the left-hand columns of the periodic table were found to be more effective suppressors than those in the center, the amounts required for a detectable effect averaging about 10 and 50 micrograms, respectively. Presence of the effect in a sample analysis can be established by examination of internal standard lines for suppression, but the magnitude is somewhat difficult to judge visually and analyses were therefore restricted to safe amounts of sample. It will be seen that there is no advantage in loading the electrode with as much sample as it can hold, since for large amounts the increased suppression amounts to as much as the increase in sample.

A further restriction on the sample size is imposed for elements with very complex spectra such as uranium and the rare earths. The rich line spectrum and continuous background limit the useful amount to about 20 micrograms for the dispersions employed.

Increasing hydrochloric acid concentration increased the intensity somewhat, but the effect becomes constant above about 0.5 percent, and one percent was therefore adopted as the standard concentration. Higher amounts do no harm except to increase the tendency of the solution to run over the sides of the electrodes during evaporation. Nitric acid decreased the intensity markedly, the tolerance being about 0.02 percent for a volume of 0.1 ml. Sulfuric acid also suppresses and the tolerance is much lower.

Since the visual comparison method depends on reproducibility of the absolute intensities, rather close control is required. For example, machined electrodes should not be allowed to stand unused for more than a day because of oxide formation. The most important factor affecting the reproducibility is probably the manner in which the solutions are evaporated on the electrodes. The distribution of the deposit over the surface varies with the temperature and with the drop size. Another factor which is difficult to control is photographic. High humidity markedly reduces the speed of the plates used, and both heating and cooling coils are required in the spectrograph room to maintain a relative humidity of less than about 30.

Portions of standard plates are reproduced in Fig. 2.

QUANTITATIVE ANALYSIS BY DENSITOMETRY

Satisfactory working curves, linear and with a slope of 45°, are obtained by the conventional internal standard method. The reproducibility obtained depends somewhat on the procedure followed, being greater if a spectroscopic buffer is added. For example, an average deviation from the mean of about 3 percent was obtained for aliquots of a uranium solution using 100 micrograms of lead as buffer and internal standard, while an average deviation of about 8 percent was obtained for a number of cases in which 4 micrograms of molybdenum was used as internal standard with no buffer. The increased precision is gained at the expense of a corresponding decrease in sensitivity by suppression. Plate calibration was made by step sector with a d.c. iron arc and the lines were corrected for background.[6]

The useful lower limits reached by densitometry are about an order of magnitude higher than by the method of visual comparison because it is difficult to measure small intensity differences between line and background with grainy plates, and because of suppression if a buffer is used. The useful upper limit approaches a milligram. The range which can be covered near the lower end with a given line and using a given set of exposure conditions is rather limited because of the high background and, especially toward the visible, the high plate contrast.

The range limitation is inconvenient for a laboratory receiving widely varying samples in numbers insufficient to warrant calibration of a number of working curves covering successive ranges for the same element. Attempts were therefore made to use a step weakener for large amounts so that the same working curve for a sensitive line would serve over a thousand-fold range.

A seven step factor-of-two sector, for example, introduces a factor of 64 in intensity reduction which, combined with the latitude of the plate and the fact that the lower amounts do not con-

[6] W. C. Pierce and N. H. Nachtrieb, Ind. Eng. Chem. Anal. Ed. **13**, 774 (1941).

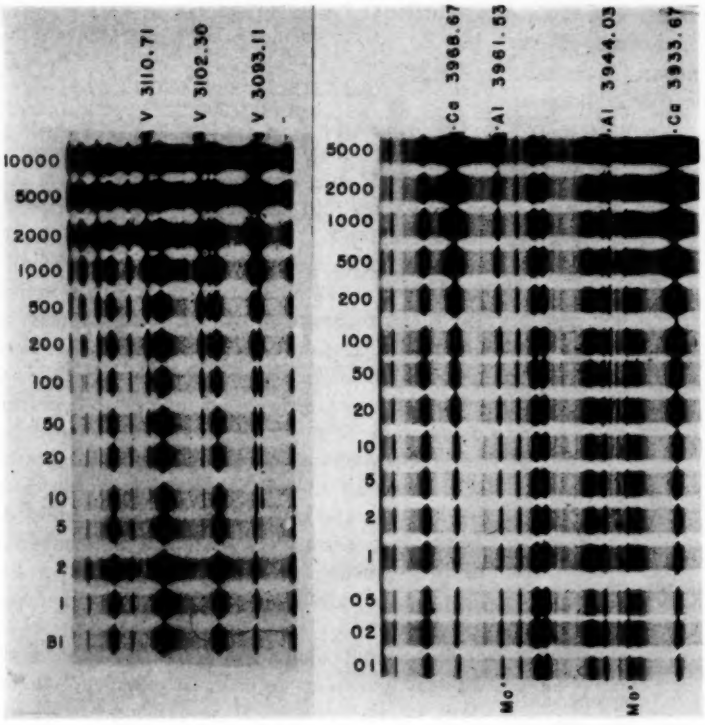

FIG. 2. Portions of standard plates for visual comparison. Units in milligammas. The calcium series is from an early plate and shows contamination at the low end and suppression of the molybdenum internal standard at the high end.

tribute a proportionate density because of the high background intensity, should be sufficient. A sector has been used successfully in this way with other light sources but is not suitable with the copper spark method. This is because of the intensity decrease with time which limits the exposure time for the line, regardless of whether a longer exposure is used to bring up the background. Moreover the spark emits only for a very small fraction of each half cycle of the 60-cycle supply so that a non-synchronized rotating sector observed in the light of the spark is seen in a number of random, shifting positions. Hence the transmission of a step is the probability that it will be open at the time of the flash. For the sector referred to above the smallest step will transmit only about two flashes per second on the average. Since most of the light is emitted in the first 10 to 15 seconds using the Baird power supply the actual intensity transmitted by the smallest step varies from exposure to exposure due to beats, statistical fluctuations due to the small number of flashes, and the relative number of early and late flashes transmitted. These conclusions were verified by comparing the reproducibility at different power levels and adjusted exposure times, by which it was shown that the average deviation of the intensity ratio of lines compared in the largest and smallest steps could be halved using low power and a correspondingly longer time. A two-minute exposure time with one microfarad and no added inductance with the Multisource was fairly satisfactory.

Since standard plates for visual comparison had previously been prepared using the Baird power supply and a 30-second exposure, it was desirable to have a weakener which could be used under the standard conditions so that the visual method could be used for those elements for which working curves were not yet available. Since a rotating sector was not satisfactory and a step diaphragm difficult to use with an astigmatic spectrograph, several attempts were made to prepare a step wedge. Suitable facilities for evaporating metallic films on quartz were not available and as a temporary expedient wedges

were made of fine grained photographic plates such as Eastman Spectrum Analysis No. 1. It was found that calibration of these wedges with a spectrophotometer did not agree with the calibration obtained with the spectrograph against a step sector using a d.c. arc. The discrepancy was ascribed to scattering by the granularity of the wedge and was especially serious because the stigmatic position for the wedge with an Eagle mounting was quite a distance from the slit. Moreover the densities of successive steps were not additive so that even calibration against a sector was not satisfactory. The scattering difficulty was eliminated by substituting Eastman high resolution plates type 548–0 for SA-1 plates for the wedge. Spectrophotometer and sector calibrations were identical for these wedges, but unfortunately they showed a yellow stain which had an absorption spectrum with considerable structure, so that the step ratios had to be calibrated at each wave-length. It would appear that commercial evaporated wedges would be welcome for use with light sources such as the copper spark having little self-absorption, and these difficulties have been enumerated in order to encourage such development.

A typical working curve obtained without a step weakener is shown in Fig. 3.

CONTAMINATION

The sensitivity inherent in the copper spark method is greater than can be used to advantage for such common elements as calcium, magnesium, aluminum, sodium, and so on, unless drastic precautions are taken to eliminate contamination. The first standard plates made for these elements showed high and erratic blanks, the lines being quite black. After the laboratory rooms were reconditioned by sealing the walls and windows, providing filtered air at positive pressure to the outside, installing generous cabinet space so that bench tops and shelves could be kept clear of equipment not actually in use, requiring personnel to change shoes on entering, and arranging for daily floor mopping and vacuum cleaning, the blanks dropped by a large factor to constant values. The residual impurities were obviously caused by reagents and volumetric ware, and an intensive system of purification and cleaning was developed. Water was redistilled and stored in quartz or tin-lined vessels. Hydrochloric acid was prepared by passing hydrogen chloride gas through suitable scrubbers into the purified water, or more conveniently by distillation through a four-foot glass packed column after considerable refluxing. The lowest blanks were obtained by diluting the stock solutions in calibrated quartz test tubes, stirring with quartz rods. The quartz ware was cleaned by soaking first in a mixture of nitric and sulfuric acids and then in hydrochloric acid, followed by rinsing with hot distilled water and finally by the pure water. Tap water is to be avoided. For less common elements the dilutions can be made more conveniently using graduated Pyrex mixing cylinders, and it was found that after these had been cleaned for many months by the above procedure the contamination introduced for common elements dropped to almost the quartz level. Zero blanks were never obtained for common elements but were usually insignificant for practical analyses. It is apparent that air-borne contamination from other laboratory operations involving dry chemicals would be serious.

APPLICATIONS

Any sample can be analyzed which can be obtained in the form of a hydrochloric acid solution with appropriate dilution or concentration

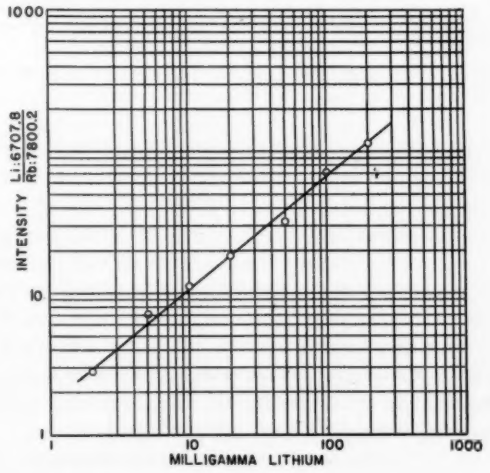

FIG. 3. Working curve for lithium using rubidium as internal standard.

to bring the amounts within the proper range. The method is ideal for the determination of impurities in acids and other liquids for which the bulk of the sample can be removed by evaporation. It can be used to determine minor impurities in any solid sample soluble in hydrochloric acid, the concentrational sensitivity being limited by the suppression or spectral interference from the main constituent, with the great advantage that to these limits no special calibration is necessary for different matrices. The concentrational sensitivity can frequently be increased by chemical separation of the main constituent.

ACKNOWLEDGMENT

The present work was developed primarily for the analysis of plutonium samples at the Metallurgical Laboratory of the University of Chicago and is the result of the cooperation of many individuals. We should like to acknowledge our primary indebtedness to Professor G. K. Rollefson, Department of Chemisty of the University of California at Berkeley, who first demonstrated the sensitivity which could be obtained, using silver electrodes of somewhat different shape from those we finally adopted. This work was greatly expanded at Chicago and subsequently at other laboratories associated with the Metallurgical Project. Substantial contributions were made by Dr. Raymond G. Russell and Dr. Wayne W. Marshall. Miss Irene Corvin made most of our standard plates. Thanks are due also for the suggestions of the analysts who used the method during its evolution.

JOURNAL OF THE OPTICAL SOCIETY OF AMERICA VOLUME 37, NUMBER 4 APRIL, 1947

A Method for Measuring the Spectral Energy Distribution of Low Brightness Light Sources

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(Received July 12, 1946)

A method is described by which it has been found possible to determine satisfactorily the spectral energy distribution for luminous surfaces whose brightness is only a few footlamberts. A standard spectrometer is used in conjunction with a photo-multiplier tube. Relative energy is found by direct comparison with a tungsten lamp calibrated for color temperature. Results obtained with this arrangement for various fluorescent lamps are in good agreement with measurements made on a large double monochromator, indicating that the scattered light in the single prism instrument does not introduce serious errors.

As long as the light sources which were commonly used were of the incandescent type, the problem of finding the energy distribution in the spectrum was largely a matter of obtaining the color temperature. When this was known there were available well established tables giving the radiated energy as a function of wave-length. With the use of fluorescent materials and phosphors, however, it is now possible to produce light sources of an unending variety of colors with corresponding spectral distributions of every sort. To compare such sources and to standardize any particular type requires the spectral energy distribution curves, and consequently it has become a matter of considerable interest and importance to obtain such curves in as easy and rapid a fashion as possible.

The chief difficulty encountered in doing this arises from the fact that the energy to be measured after the light is dispersed into a spectrum is extremely small. It is not an easy measurement to make even for a concentrated incandescent source, where the amount of light accepted by the spectrometer is relatively large. Fluorescent sources are generally of low brightness and the amount of light that can be brought through a suitable dispersing system is limited to very low values.

Recent improvements in photoelectric cells of the electron-multiplier type have made it possible to work satisfactorily at these low light levels. A number of reports have appeared lately which tell about the use of these cells for comparing the intensity of spectral lines in spectrochemical analysis. Several of these are listed below.[1–4] The purpose of this note is to describe how a cell of this kind may be used with a good quality portable spectrometer to determine the spectral energy distribution for low brightness sources which have a continuous spectrum. Using a conventional constant deviation spectrometer it is a relatively simple matter to make satisfactory measurements on a source whose surface brightness is no more than 3 or 4 footlamberts. With a short focus concave grating it is possible to work with sources of considerably lower brightness. The limiting factor is generally photo-cell noise due to thermionic emission, and this may be greatly reduced by keeping the cell at dry ice temperature.

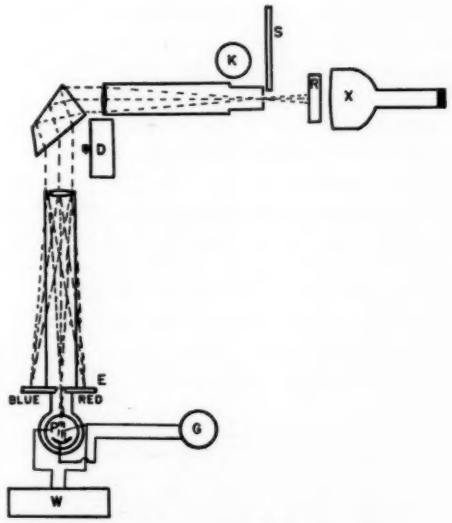

FIG. 1. Arrangement for measuring spectral distribution.

[1] D. H. Rank, R. J. Pfister, and P. D. Coleman, J. Opt. Soc. Am. **32**, 390 (1942).
[2] D. H. Rank, R. J. Pfister, and H. H. Grimm, J. Opt. Soc. Am. **33**, 31 (1943).
[3] G. H. Dieke and H. M. Crosswhite, J. Opt. Soc. Am. **35**, 471 (1945).
[4] G. H. Dieke, Research Report No. W-193, Office of Production Research and Development, War Production Board.

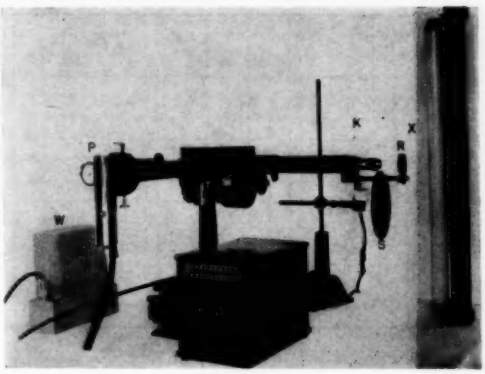

FIG. 2.

EXPERIMENTAL ARRANGEMENT

The arrangement used is indicated in Fig. 1, and a photograph of the actual equipment is given in Fig. 2. Light from the source (X) being measured and from the standard lamp (K) is admitted alternately to the spectrometer by means of a shutter (S) and the magnesium carbonate reflector (R). S and R are so connected that when the light from X is incident on the spectroscope slit, the shutter S keeps the light of the standard lamp from falling on the source X.

The light in a given wave-length band determined by the setting of the exit slit E, falls on the photo-cathode of the 9-stage electrostatically focussed multiplier P, on which a d.c. voltage of 50 to 100 volts per stage is maintained by means of a well regulated supply. The output of the photo-cell may be read directly on a galvanometer (an instrument with a sensitivity 5×10^{-9} amp./mm with an Ayrton shunt was used in this work) or it may be fed into a d.c. amplifier so as to obtain readings on a microammeter.

The voltage on the multiplier is adjusted to give galvanometer deflections of suitable magnitudes at the wave-length region being studied. Then galvanometer deflections are taken for the standard lamp (d_k) and for the unknown (d_x). The relative energy of the unknown can then be computed simply as $d_x R \cdot r/d_k$ where R is the relative emission of the known lamp and r is the reflectance of the $MgCO_3$ block at the wave-length under consideration.

It has been found desirable to reflect the light of the standard lamp from a diffuse surface such

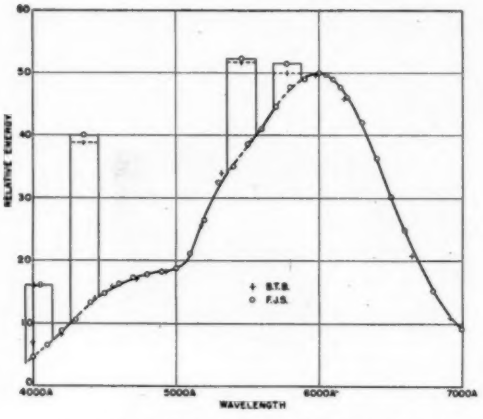

FIG. 3. Spectral distribution of radiation from 40-watt F lamp No. B-36 3459.

as magnesium carbonate to be sure that the aperture of the spectrometer is filled with light in the same way as it is from the unknown source. This insures that the same part of the photocathode is used for both sources, and that the absorption of the lenses and prism is also the same. Magnesium carbonate is easy to obtain and clean, but since its properties vary somewhat from specimen to specimen, a calibrated white tile such as those obtainable from the Bureau of Standards is preferable.

The standard source used was a 100-watt gas-filled tungsten lamp calibrated at a color temperature of 2848°K, and operated at the specified current and voltage.

The width of the exit slit can be adjusted by an external screw and its dimension determined by a dial indicator. In making measurements on a continuous spectrum it is not necessary to know the width of the wave-length band which is transmitted, since this is the same for both the standard and the unknown. With low intensity sources it is desirable to use as wide a spectral band as possible. This is of course limited by the steepness of the slope of the energy distribution curve. If there are sharp peaks in the energy curve, it is necessary to use a narrow wave-length band.

When a line spectrum is superposed on the continuous background, such as is generally the case with fluorescent lamps, the energy in any line can be found by setting on the line and calculating the energy value in the same way as for any point in the continuous spectrum. The amount of the continuous background can be estimated closely from points taken on either side of the line. When this is subtracted from the measured value at the line, the energy in the line is obtained. It has become customary to represent this energy on the distribution curve as though it were spread over 200A. Thus, if the exit slit is set to let through 50A the value calculated as above would be divided by 4, and it would be indicated as a uniform energy band 200A wide. This is illustrated for the case of a fluorescent lamp in Fig. 3. It is necessary, in this procedure, to make an accurate determination of the wave-length band which the exit slit allows to pass. This may be done by making wave-length settings first on one edge of the exit slit and then on the other, for the line being studied, and noting the wave-length band covered. It may also be determined by choosing properly located lines, and measuring the linear dispersion directly with a traveling microscope. When the dispersion curve of the prism has once been determined, it is a simple matter to calculate the wave-length band transmitted at any part of the spectrum if the width at any one wave-length has been measured.

The regulated 1200-volt supply used is a radio-frequency unit designed by Mr. S. Hansen and Mr. R. Frankland of this laboratory, the details of which are to be described in a subsequent note. The unit has the advantage that short circuit current is approximately 5 milliamperes, so that there is little danger of serious shock.

Typical curves obtained by this method are shown in Figs. 3–5. Figure 3 was taken for a fluorescent lamp. The relative energy in the

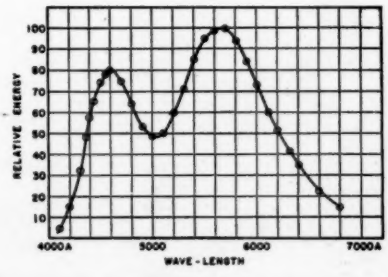

FIG. 4. Spectral distribution of light from television tube type 10BP4 brightness 12 foot lamberts.

mercury lines is indicated by the area of the vertical columns. Figure 4 gives the spectral curve for the screen of a television tube operated at a brightness of 6 footlamberts. In Fig. 5 is shown the energy distribution of light from the north sky, taken by pointing the spectroscope out of a fifth-story window. This was taken with a band width of approximately 6A at 4000A and 45A at 7000A. No attempt was made here to record the fine structure which was evident as the spectrum was moved across the photo-cell.

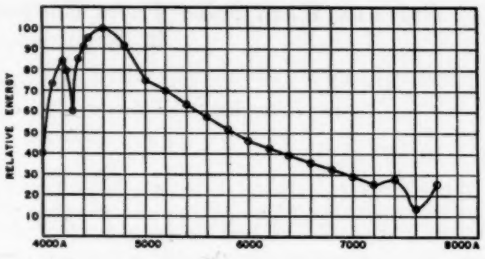

FIG. 5. Spectral distribution of light from north sky 12:00 noon, May 3, 1946 (Schenectady, New York) (slight haze).

ACCURACY ATTAINABLE

The accuracy which can be attained using the method described is of course limited by a number of factors, the most important being (1) scattered light in the spectrometer and (2) calibration of the standard lamp and reflector assembly. On account of the former there was considerable question as to the possibility of working with a single-prism monochromator of the type used. However, when a check was made with Dr. B. T. Barnes of Nela Park, who used a carefully calibrated double monochromator, the results agreed surprisingly well. Spectral curves were taken for a series of fluorescent lamps of different spectral distribution, and the maximum discrepancy at any point in the continuous spectrum was approximately 3 percent, and for some lamps the agreement was much closer than this throughout the spectrum. Since neither set of measurements had been taken with the aim of ultimate precision, this seems to indicate that the error due to scattered light in the single-prism instrument is not large. The distribution curve for one of the lamps which was compared in this way is given in Fig. 3.

ADVANTAGES OF THE METHOD

The method has two notable advantages:

(1) It requires standard equipment, for the most part, and is relatively simple.

(2) Energy measurements are made in terms of a standard lamp and are almost independent of photo-cell characteristics, as well as any selectivity there may be in the optical system. In addition, it should be noted that errors due to poor adjustment of the optical system will tend to affect the standard and unknown in the same way, and consequently will be minimized in the result.

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Notes on W. S. Stiles' Paper Entitled, "A Modified Helmholtz Line-Element in Brightness-Colour Space"*,[1]

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(Received December 14, 1946)

A brief summary is given of the paper by Stiles. The line-element suggested by Stiles is shown to be the sum of the squares of total differentials of three magnitudes. Therefore, the corresponding color threefold is Euclidean. For the same reason, the line-element originally proposed by Helmholtz also defined a Euclidean color space. The customary parameters of colorimetry correspond to curvilinear coordinates in the color space of both Helmholtz and Stiles. In both of those spaces, constant luminosity is represented by curved surfaces. The forms of the line-elements of both Helmholtz and Stiles imply that geodesics, representing series of colors which exhibit the least number of just-perceptible differences between any two colors, are straight lines in the Euclidean spaces defined by those line-elements. They also imply that ordinary length measured along these lines is proportional to the number of just-perceptible differences between the colors represented by the terminal points. No direct tests of these implications are known. MacAdam's data on the discrimination of equiluminous colors determine a surface of constant luminosity in the color discrimination space for at least one observer (PGN). This surface exhibits such severe (and irreducible) curvature that the straight lines drawn between most pairs of considerably different equiluminous colors are, at their midpoints, far from the surface of constant luminosity. The line-elements of both Stiles and Helmholtz, therefore, imply that the series of colors exhibiting the least number of just-perceptible differences between two equiluminous colors includes many colors of very different luminosity. Although no observational tests are known, it seems unreasonable to expect that considerable departures from the common luminosity of the terminal colors will reduce the number of differences perceptible in any series between them. Attention is called to the criterion of probability of errors in color matching as an alternative to just-perceptible differences for investigations of visual sensitivities to color differences. This concept leads to a definition of the line-element independently of any assumption of the applicability to color discrimination of Fechner's law or its modifications.

STILES begins by recalling Von Helmholtz' line-element (δs), of 1891. If x, y, z and $x+\delta x$, $y+\delta y$, $z+\delta z$ be the trichromatic coordinates of any two "just-distinguishable" patches, then (apart from the factor $\frac{1}{3}$ introduced by Stiles for convenience), Von Helmholtz' relation, sometimes referred to as *the threshold condition*, is

$$\delta s^2 \equiv \frac{1}{3}\left\{\left(\frac{\delta x}{a+x}\right)^2+\left(\frac{\delta y}{b+y}\right)^2+\left(\frac{\delta z}{c+z}\right)^2\right\}=F^2, \quad (1)$$

where a, b, c, F are constants.

After mentioning a fair success of this line-element with the suite of spectral colors, as measured for their trichromatic coordinates by Arthur König, and pointing out some serious shortcomings of (1), Stiles quotes the alternative line-element, proposed by Schrödinger,

$$\delta s^2 \equiv \frac{\alpha\delta x^2/x+\beta\delta y^2/y+\gamma\delta z^2/z}{\alpha x+\beta y+\gamma z}=F^2, \quad (2)$$

where α, β, γ, F are constants, with the implication that *brightness* is directly proportional to the linear function $\alpha x+\beta y+\gamma z$; so that, for equiluminous colors, (2) reduces to a Euclidean element. Without giving an adequate criticism of (2), Stiles finds it inconsistent with threshold measurements considered by Stiles in the body of his paper.

The attempts of "several investigators" more recent than Schrödinger's are mentioned only summarily. According to Stiles, the implied measurements were made at rather high intensities and only for the equiluminous (bi-dimensional) sub-domain of the complete domain of colors. Stiles' "modified" line-element is based on observations of a different nature which (in his own opinion) are best introduced by contemplating the application of the Helmholtz element to another special class of just-distinguishable pairs, *viz.*, of liminal-increment type.

Such is the subject of Section 2 of the paper under discussion. By definition, a pair is of the last-mentioned type if "the physical stimulus in one patch is obtained from that of the other by

* Communication No. 1132 from Kodak Research Laboratories.

[1] W. S. Stiles, Proc. Phys. Soc. London **58**, 41–65 (1946).

the *addition* of a further stimulus," as, e.g., when one patch being monochromatic of wave-length μ and energy W_μ, the other is a mixture (additive) of wave-lengths μ and $\lambda \neq \mu$ and energies W_μ and U_λ. Formula (1) becomes, in this case,

$$\left[\frac{U_\lambda r_\lambda'}{a+W_\mu r_\mu'}\right]^2+\left[\frac{U_\lambda g_\lambda'}{b+W_\mu g_\mu'}\right]^2 +\left[\frac{U_\lambda b_\lambda'}{c+W_\mu b_\mu'}\right]^2=3F^2, \quad (3)$$

where r_λ', etc., are the fundamental response functions of the x, y, z, or—as Stiles puts it—the "red, green, blue mechanisms," respectively. He then gives a simple, formal, interpretation of the expression (3) which need not be rewritten here.

Section 3 is devoted to a detailed discussion of "basic measurements" guided by (3) and, following the hints given by these, Stiles constructs, in Section 4, his own "modified line-element," *viz.*, as a *tentative* element "for application to *any* pair of just-distinguishable light patches," not only those of the liminal-increment type.

Let the patches P and P' have the absolute energy distribution $W_\lambda d\lambda$ and $W_\lambda' d\lambda$. For brevity, put $\zeta(p)=9\xi(p)$ [see Section 3],

$$R=\int W_\lambda R_\lambda d\lambda, \quad \delta R=\int (W_\lambda'-W_\lambda)R_\lambda d\lambda,$$

and similarly for G, δG; B, δB, where $\xi(p)$, R_λ, G_λ, B_λ are the experimental functions plotted (before), in Stiles' Figs. 2–5.[2] Further, let ρ, γ, β be proportional to the Fechnerian limiting fractions of the three "mechanisms," i.e.,

$$\rho:\gamma:\beta=\frac{R_\lambda}{r_\lambda}:\frac{G_\lambda}{g_\lambda}:\frac{B_\lambda}{b_\lambda}=0.78:1\ .4.46,$$

satisfying, moreover, the condit n

$$\frac{1}{\rho^2}+\frac{1}{\gamma^2}+\frac{1}{\beta^2}=1$$

so that

$$\frac{1}{\rho^2}, \frac{1}{\gamma^2}, \frac{1}{\beta^2}=0.612,\ 0.369,\ 0.018_5.$$

[2] The definitions of R_λ, etc., are modified in the sequel.

Then Stiles' proposed element is

$$\delta s^2 \equiv \left[\frac{1}{\rho}\zeta(R)\delta R\right]^2+\left[\frac{1}{\gamma}\zeta(G)\delta G\right]^2 +\left[\frac{1}{\beta}\zeta(B)\delta B\right]^2=F^2, \quad (4)$$

where $F=$constant, as before. At *high* intensities, this reduces to the simpler form,

$$\delta s^2=\left[\frac{1}{\rho}\frac{\delta R}{R}\right]^2+\left[\frac{1}{\gamma}\frac{\delta G}{G}\right]^2+\left[\frac{1}{\beta}\frac{\delta B}{B}\right]^2. \quad (4a)$$

[To facilitate comparison with Stiles' paper, notice that (4) and (4a) are Stiles' formulas (13) and (14).] The following sections of the paper are devoted to the application of the "modified line-element" to the questions of color matching, step-by-step visibility curve, and so on.

At this stage, however, let us stop at formula (4) itself, with its high intensity reduction, (4a). Consider the complete three-dimensional color domain, briefly, the *threefold* R, G, B, that is to say, with triples (R, G, B) as its elements or, figuratively, its *points*.

Whatever the functions $\zeta(R)$, $\zeta(G)$, $\zeta(B)$ of their arguments (R, etc.), $\zeta(R)\delta R$, etc., are total differentials (or "variations") of some variable magnitudes, and since ρ, γ, β are mere numerical constants, in fact, 0.612, etc., such also is the case of $(1/\rho)\zeta(R)\delta R$, etc., say,

$$\frac{1}{\rho}\zeta(R)\delta R=\delta u, \quad \frac{1}{\gamma}\zeta(G)\delta G=\delta v, \quad \frac{1}{\beta}\zeta(B)\delta B=\delta w. \quad (5)$$

Thus, Stiles' element becomes

$$\delta s^2 \equiv \delta u^2+\delta v^2+\delta w^2=F^2, \quad (6)$$

no matter what the "intensities" (high or low).

That is to say, the color threefold is simply *Euclidean* or, in Clifford's phraseology, *homaloidal*, and can, therefore, be faithfully portrayed in ordinary Euclidean three-space. (Such, of course, was also Von Helmholtz' domain of 1891, though my old, and much revered, teacher failed to utilize this simple property consistently.)

Needless to say, if the color threefold has this property (of being homaloidal or "flat," if one so prefers) with our u, v, w as coordinates of a point (color), it will enjoy this property with any other

system of coordinates. In fine, homaloidalism is an *intrinsic* property of a manifold, invariant with respect to any coordinate transformations whatever, linear or not.

Strangely enough, Stiles, as also his famous predecessor of fifty-five years ago, does not make any use of this simple property. Yet, its implications are obvious and, at the same time, of prime importance for the theoretical as well as the experimental colorimetrist.

Let us, then, consider here these implications. They are trivially simple.

First of all, the color threefold being (according to Stiles' proposed element) homaloidal, its *geodesic surfaces* are *planes, viz.*,

$$\alpha u+\beta v+\gamma w=\text{const.},$$

where α, etc., are constants. The (Gaussian) *curvature* of any such surface (twofold) is just *zero*.

Again, the *geodesics* or *shortest color lines* are intersections of pairs of such surfaces, planes, or, simply: the color geodesics are *straight lines*, just the straight lines of a space-model with, say, u, v, w as Cartesian coordinates, and the ordinary length measured along these straights is strictly equal (or proportional) to the color-distance of any two colors of the straight line as representative of the contemplated suite of colors.

Now, these idyllically simple relations or properties are emphatically at variance with experience, especially with the results of tens of thousands of color matchings made recently by MacAdam, with Nutting and others as "observers." But, if so, then Stiles' painstakingly constructed "modified" Helmholtz element falls to the ground.

Let us try to substantiate this general impression of utter inadmissibility of Stiles' line-element.

If MacAdam's careful and extensive measurements (color matchings) pertained to the full, unrestricted color threefold, no such "substantiations" were needed. Unfortunately, however, they have been (so far) limited to a twofold (or twofolds) of *equiluminous* colors, and this circumstance necessitates some detailed discussion.

Let us confine our attention to some prescribed, constant luminosity. Then, the equiluminous color twofold is isomorphous with a certain, completely fixed surface. Now, this surface has (through MacAdam's investigation in these Laboratories) turned out not only to differ emphatically from a plane, but as well from any surface of constant curvature. Its curvature varies from point to point (color to color) in a highly complicated, well-nigh fantastic, manner and does not even retain its sign, being positive in some quarters (of the "Maxwell triangle") and negative in others. Its model in paper,* far from resembling a plane or a sphere, pseudo-sphere, or any of the comparatively tame surfaces ever studied and formulated by geometers, looks rather like some monstrous mushroom. One would try in vain to write down (*in toto*) its equation.

Such being the case, let P and Q be any two points (colors) chosen on this equiluminous twofold or surface, which will be briefly referred to as a "Nutting surface." According to Stiles' proposed line-element, the shortest line (containing, that is, the least number of "unit steps," just-perceptible steps) is the Euclidean *straight line*, PQ, and their color-distance is just the length, $\langle PQ\rangle$, of this straight segment. Now, the straight PQ deviates (in general) most emphatically from the equiluminous surface, the Nutting surface.[3] In other words, the actual shortest line from P to Q (of same luminosity) would be a locus of colors differing considerably in luminosity from both end-colors, P and Q. Is this credible?

We do not know whether or not experiments have been purposely made to decide this question, *viz.*: Does the shortest passage from one to another color of equal luminosity lead through colors of much different luminosity? The answer is, of course, not a matter of opinion, but of fact, and we must leave it to technical experts on colorimetry, although, *prima facie*, the answer would seem to be in the negative and thus against Stiles' line-element.

In the meantime, this element can be definitely condemned or ruled out by another reasoning. Namely, Stiles himself derives, in his Appendix, p. 64, for the Gaussian curvature of the *equiluminous* surface (or "surface of constant C.I.E.

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* Reproduced in J. Frank. Inst. **238**, 203 (1944).

[3] And it does not seem likely that it would deviate less from surfaces constructed from color matchings by other observers.

brightness"), from his modified line-element the expression,

$$K=\frac{\rho^2\gamma^2\beta^2(1+9R)(1+9G)(1+9B)v_r v_g v_b(9y'+v_r+v_g+v_b)}{\text{squared sum of three squares}}$$

Formula 30), where v_r, v_g are positive, while v_b is negative, yet so that $V_r+V_g+V_b$ is positive. Consequently, the curvature K is *always negative, viz.*, for any C.I.E. brightness. Now, the Nutting surface shows here a pronounced positive and there an equally pronounced negative curvature, with transitions through flatness. In fact, Stiles himself candidly states (p. 65) this discrepancy or inconsistency of his modified line-element "with MacAdam's results for P.G.N." (Nutting, as observer). Nay, he concludes his paper by adding that this inconsistency persists, whatever the choice of ρ, γ, β, and so on. No wonder, for K is invariant with respect to any coordinate-transformations whatever.

Thus, unless some other observers will lead to equiluminous surfaces of throughout negative curvature, Stiles' element stands condemned, in spite of some of its successes noted in the body of his paper.

Independently of this specific objection, we should like to draw the reader's attention to the fact that his fundamental concept of pairs of "just-distinguishable" color patches and the associated unit-steps (not perceptible), taken over from Von Helmholtz and König, is rather vague. At any rate, MacAdam's and our own concept of "standard deviation" in color matchings, paired with the rather plausible distribution function of probabilities of the matches, seems to offer a more sound, and less arbitrary, foundation for the metrics of the color domain.[4] In the latter treatment, the "line-element," δs^2 or ds^2, is not introduced *ab initio*, but appears only as an implication of the matching probabilities, *viz.*, as the (negative d) exponent, $g_{ik}\,dx_i\,dx_k$ (summed over $i, k=1, 2, 3$), of the expression for the probability function, and with the most probable values of the six coefficients, g_{ik}, determined jointly by all matchings of any, and every, central color chosen within the whole color domain.

It is regrettable that Stiles, who seems very conscientious and thorough in judging the attempts of other workers, did not comment upon our alternative criterion of color discrimination, based fundamentally on matching probabilities. It may be that this omission resulted from a wartime delay in transatlantic shipment of the January, 1945, issue of the *Journal of the Optical Society of America*. We would be interested in Stiles' opinion of our criterion in comparison with the rather objectionable concept of pairs of "just-distinguishable patches" which bids him to construct the element, δs^2, by sheer guesswork, actually by a slight modification of Von Helmholtz' tentative guess (associated with Fechner's ideas) of 1891.

We may profitably return to all such and similar questions when the free, three-dimensional color matchings, now in preparation by MacAdam, will be available. For these will supersede (and possibly shed some instructive light upon) his previous results hitherto limited to the exploration of twofolds of equiluminous colors, and thus, only of a few *sections* (so to speak) of the complete domain of colors.

[4] L. Silberstein and D. L. MacAdam, J. Opt. Soc. Am. **35**, 32 (1945); L. Silberstein, J. Opt. Soc. Am. **36**, 464 (1946).

An Infra-Red Prism Spectrometer of High Resolving Power*

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An infra-red spectrometer employing interchangeable 60°-prisms with faces 10 by 15 cm of potassium bromide, sodium chloride, and lithium fluoride in Wadsworth-Littrow mounting is briefly described. A one-meter 10° off-axis paraboloid is used for collimation. The radiation passing through the exit slit is focused on the thermocouple receiver by means of an ellipsoidal mirror with foci at distances 42 and 6 cm. The entrance slit has two sets of curved jaws, one set for the LiF prism and another for the NaCl and KBr prisms. When prisms are changed by push-button control the proper entrance slit automatically moves into position. The position of sharp bands can be determined to within ± 0.005 micron at 5μ and ± 0.002 micron at 10μ. The correction for stray radiation is less than 1 percent of full scale deflection below 13μ. With the potassium bromide prism, and lithium fluoride shutter, the correction for stray radiation is 5 percent of total deflection at 15μ and 10 percent at 18μ. When a sodium chloride shutter is used beyond 18μ the correction for stray radiation is negligible. The construction and performance of the instrument are illustrated by nine figures.

THE high resolution required for the interpretation of infra-red absorption spectra has generally necessitated the use of grating instruments. The limited spectral usefulness of a grating, the overlapping of orders, and the necessity of employing a fore-prism or other means of limiting the range of incident wave-lengths, however, lead to complexity in the construction and operation of a grating instrument suitable for general infra-red work over the entire 2 to 25μ range. The instrumental complexity is reduced when prisms are used instead of gratings, but when this is done resolution is usually sacrificed to a considerable extent. The present paper describes a versatile prism spectrometer which, although not unusual in design, has a resolving power approaching that of most grating instruments. One spectrometer has been built by the Phillips Petroleum Company, and a similar instrument has been built in the Physics Shop of the University of Oklahoma for the Naval Research Laboratory. The design and performance of the latter are briefly described in the present paper.

The spectrometer employs potassium bromide, sodium chloride, and lithium fluoride 60°-prisms of 10- by 15-cm faces in Wadsworth-Littrow mounting. A diagram of the optical parts is given in Fig. 1. A one-meter 10° off-axis aluminized paraboloid is used for collimation. An ellipsoid with foci at distances of 42 and 6 cm images the exit slit on the detector. Each of the three interchangeable prisms is mounted with a Wadsworth mirror in a separate aluminum casting on the prism table. The prisms are interchanged by rotation of the table about its center. The prism table is mounted on an arm supported by a shaft through the instrument base-plate, and wave-length scanning is effected by rotation of the entire assembly about an axis at the intersection of the plane bisecting the refracting angle of the prism and the plane of the Wadsworth mirror. This condition is fulfilled for each of the three prisms, so that maximum resolution is obtained at all wave-lengths. When a prism is in its middle position a straight line from the scanning axis to the midpoint of the front prism face is perpendicular to the parallel beam of radiation coming from the parabolic mirror. This reduces the width of the parallel beam of radiation required

* Presented at the Symposium on Molecular Structure and Spectroscopy held at Ohio State University, June 10 to 15, 1946.

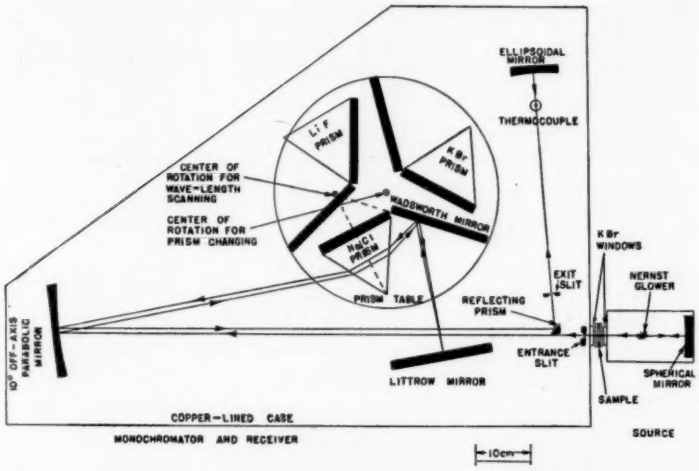

FIG. 1. Optical diagram of the spectrometer.

to fill the prisms at all wave-lengths to a minimum.

The prisms are interchanged by push-button control. Small low-speed synchronous motors automatically advance the prisms in a three-step operational cycle providing for (1) release of the prism-table locking device, (2) rotation of the prism table through approximately $\frac{1}{3}$ revolution, and (3) positioning and locking of the prism table. Locking of the prism table relative to its supporting arm is effected by a hardened steel wedge closing under constant spring tension between two steel blocks on the periphery of the table. The calibration is maintained constant to within ± 0.001 micron for repeated changing of the prisms by the automatic mechanism.

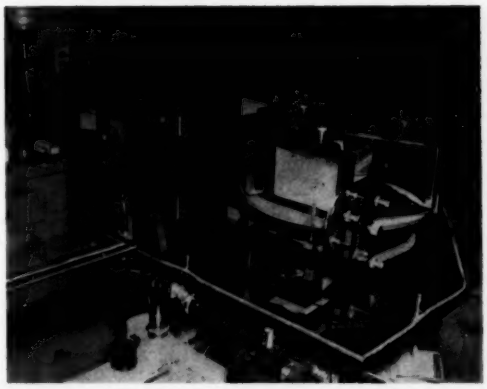

FIG. 3. Spectrometer with cover removed.

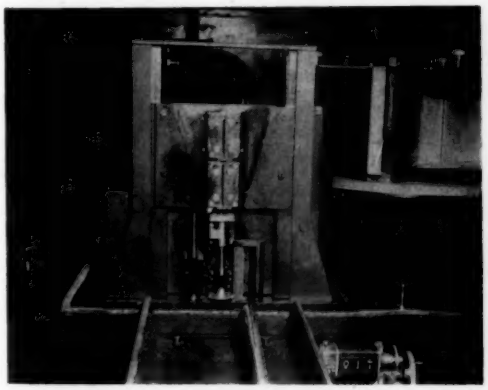

FIG. 2. Slit assembly.

FIG. 4. Spectrometer, recorder, and control panel.

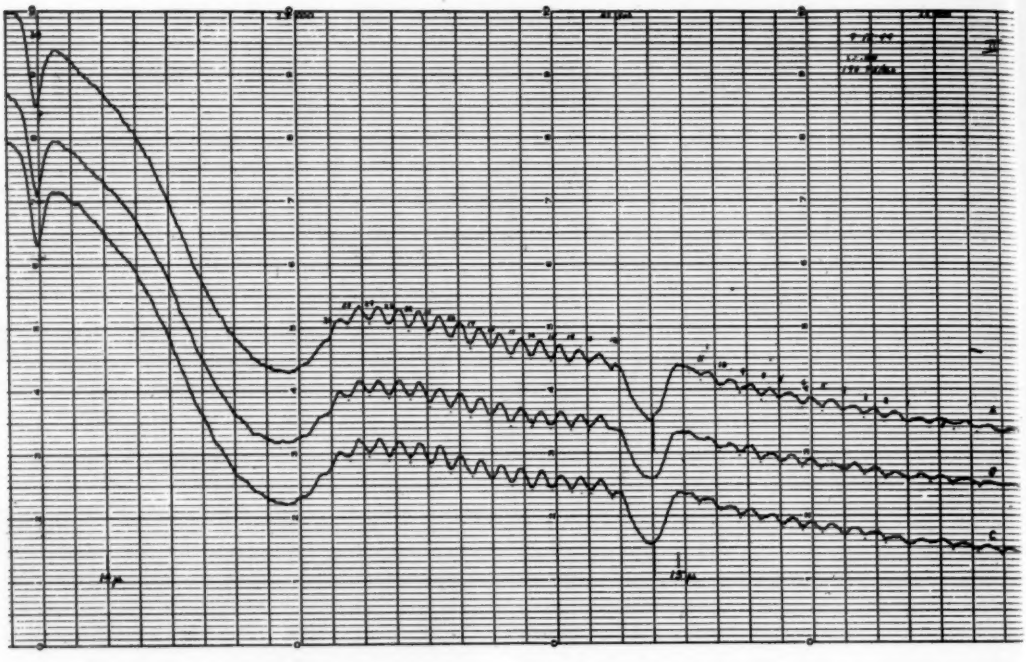

FIG. 5. The absorption of atmospheric carbon dioxide in the 15-μ region; NaCl prism.

The slits are 4 cm long and have bi-lateral action. The jaws are made of stainless steel and close under spring tension. The entrance slit has two sets of jaws, an upper having the proper curvature for use with the NaCl or KBr prisms and a lower set having the proper curvature for use with the LiF prism. An elevator automatically brings the proper slit into use when the prisms are changed. The jaws of the exit slit are straight. Both slits are operated simultaneously by manual control, and the slit-width is read directly in hundredths of a millimeter on a Veeder counter. The slit assembly is shown in Fig. 2.

A Pfund-type compensated vacuum thermocouple made by Mr. P. Weyrich at the University of Michigan is used as detector. A polythene-insulated co-axial cable in conduit connects the thermocouple to a photoelectric amplifier similar to that described by McAlister.[1] A Rubicon Type L galvanometer of period 3.8 sec. and RCA 868 photo-tubes are used. The optical components of the amplifier are mounted on a very rigid frame. The entire amplifier is enclosed in an aluminum case which is shock-mounted with rubber on an all-metal vibrationless Müller support similar to that described by Strong.[2] The amplifier output is recorded by a Leeds and Northrup Speedomax recorder. Full scale deflection corresponds to one microvolt signal from the thermocouple.

A Nernst glower in a water-cooled vacuum chamber equipped with a spherical mirror and a KBr window constitutes the source. The regulated power supply for the source will be described elsewhere. The spectrum is scanned by a synchronous motor drive and gear-box coupled to the prism table assembly by a worm and sector. Synchronization of wave-length drive and recorder chart is attained with Selsyn motors. The slits are adjusted step-wise, and duplicate traces are recorded for sample and blank placed alternately in the light beam. The atmosphere in the air-tight spectrometer case is purified by

[1] E. D. McAlister, G. L. Matheson, and W. I. Sweeney, Rev. Sci. Inst. **12**, 314 (1941).

[2] J. Strong, *Procedures in Experimental Physics* (Prentice-Hall, Inc., New York, 1943), p. 328.

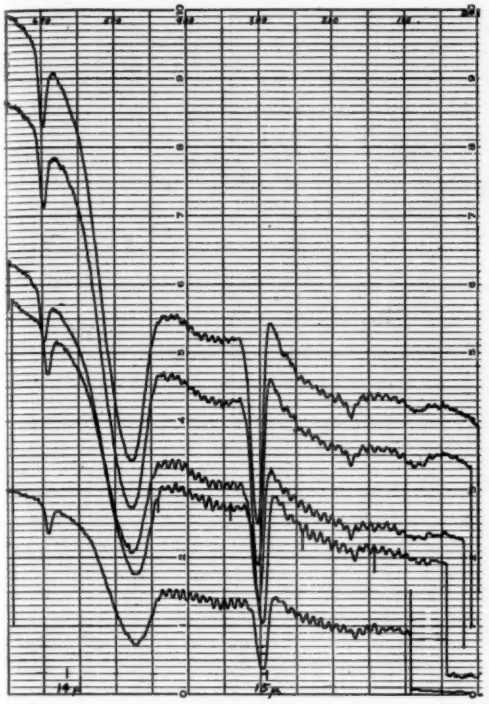

FIG. 6. The absorption of atmospheric carbon dioxide in the 15-μ region; KBr prism.

circulation through P_2O_5 and Ascarite towers. A view of the spectrometer with cover removed is shown in Fig. 3, and the complete equipment is shown in Fig. 4.

Approximately 90 minutes are required to cover the 2- to 15-μ range with the sodium chloride prism, the recording speed being limited by the 3-second response time of the thermocouple employed. In order to measure all resolvable details in the absorption curve, a six-foot length of recorder chart is required for this spectral region. A wave-length indicator on the front of the spectrometer shows the position of the instrument to 0.1 microns. The position of sharp bands can be determined accurately to within ± 0.005 micron at 5μ and ± 0.002 micron at 10μ by the aid of a table relating wave-lengths to the readings on a Veeder counter on the wave-length drive. Masks covered with black velveteen are used in several places inside the spectrometer. Metal, glass, and lithium fluoride shutters are used beyond 2, 5, and 10μ, respectively. They limit the correction for stray radiation to less than 1 percent of full scale deflection below 13μ and to about 5 percent of full deflection at 15μ.

The potassium bromide prism is used from 15 to 25 microns. A lithium fluoride shutter limits the correction for stray radiation to about 5 percent at 15μ and to 10 percent at 18μ. Use of a sodium chloride shutter beyond 18μ eliminates the effect of stray radiation nearly completely.

The performance of the instrument in the

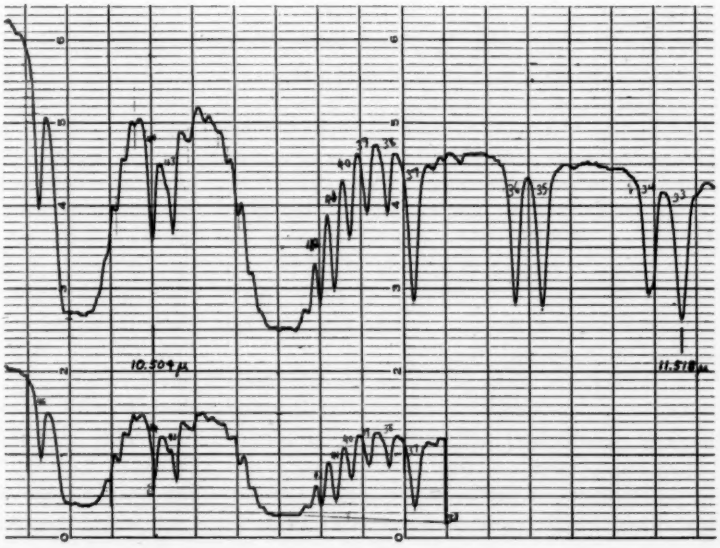

FIG. 7. The absorption of ammonia in the 11-μ region; NaCl prism.

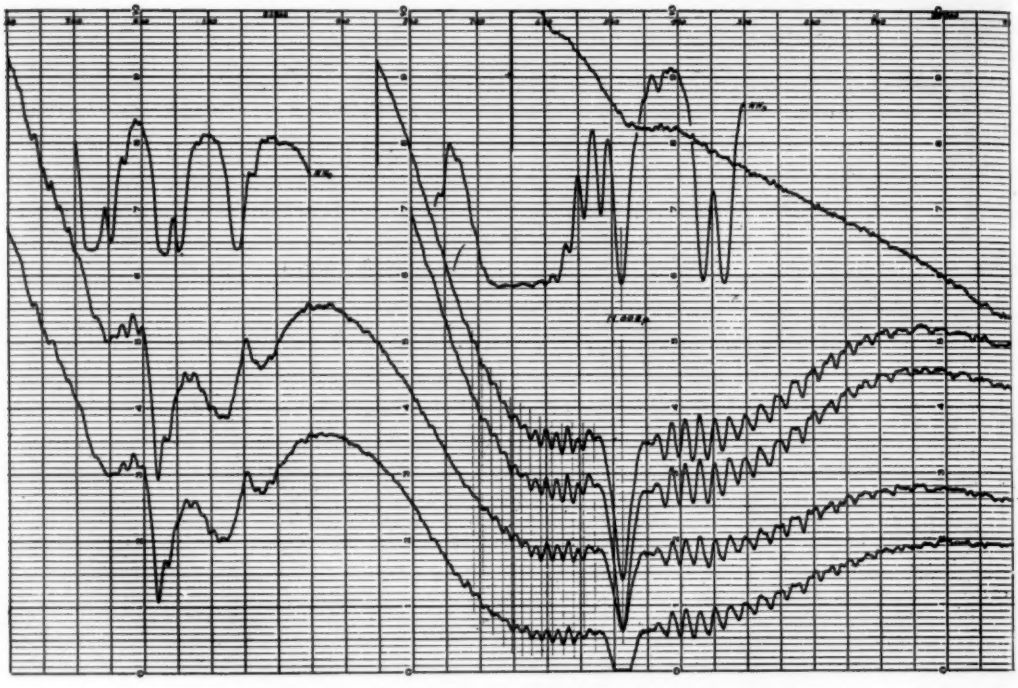

FIG. 8. The absorption of butadiene in the 11-μ region; NaCl prism.

region of 15μ is shown by Figs. 5 and 6 which give reproductions of actual records of atmospheric carbon dioxide absorption obtained with the NaCl and KBr prisms, respectively. The rotational lines are well resolved, and about equal resolution is obtained with either prism when slit-widths giving equal measurable energy are used. The broad absorption at 14.35μ is caused by an undesired film of polystyrene on the prisms. The performance of the NaCl prism is also illustrated by the recorder traces of ammonia (Fig. 7), butadiene (Fig. 8), and by selected bands in the spectra of HCl, SbH_3, and NH_3 which are plotted on a standard chart (Fig. 9). Bands as close as 25 cm^{-1} at 2.8μ, 20 cm^{-1} at 3.5μ, 5 cm^{-1} at 5μ, 1.3 cm^{-1} at 8μ, 1.1 cm^{-1} at 11μ, and 1.5 cm^{-1} at 15μ have been resolved.

Unfortunately, the large lithium fluoride prism ordered when the instrument was being designed has only recently been delivered. Use of a smaller LiF prism appeared to offer no advantage, since the performance of the large NaCl prism in the short wave-length region is as good as that reported for a smaller LiF prism with faces 8 by 10 cm.[3] It is anticipated that much better resolution will be attained in the 2- to 6-μ region when the large LiF prism is installed.

The design and construction of this instrument were based on the following considerations. First, the length of the slits, the sizes of prisms and mirrors, and the Wadsworth-Littrow mounting which allows the full size of the prisms to be utilized at all wave-lengths, were chosen to allow as much radiant energy flux as possible to pass through the spectrometer. The importance of this is indicated by the fact that equal resolution at 15μ is obtained with the NaCl and the KBr prisms (Figs. 4 and 5), although NaCl has three times the dispersion of KBr. Second, the 7 to 1 focal ratio of the ellipsoidal mirror was calculated to give the maximum signal at the thermocouple for a given signal at the exit slit obtainable with the chosen dimensions of prisms and slits. The beam of radiation converging on the thermocouple actually has an aperture ratio

[3] N. Wright, Rev. Sci. Inst. **15**, 22 (1944).

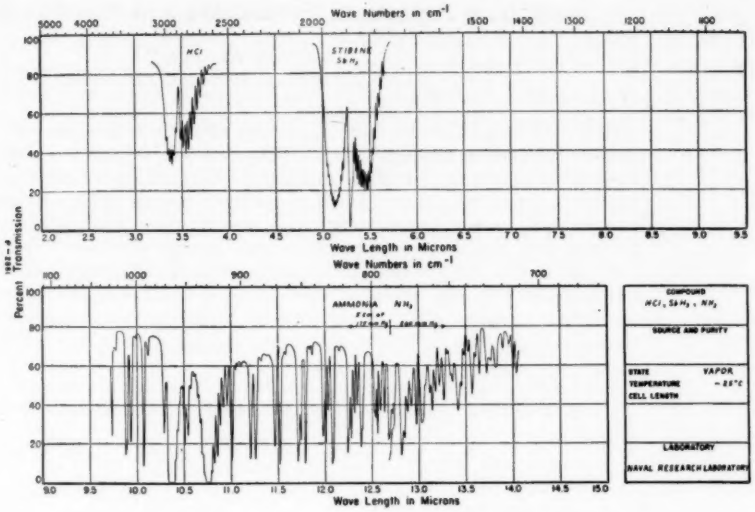
Wave Numbers in cm⁻¹
5000 4000 3000 2500 2000 1500 1400 1300 1200 1100
HCl
STIBINE
SbH₃
Percent Transmission
2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0 7.5 8.0 8.5 9.0 9.5
Wave Length in Microns
Wave Numbers in cm⁻¹
1100 1000 900 800 700
AMMONIA NH₃
9.0 9.5 10.0 10.5 11.0 11.5 12.0 12.5 13.0 13.5 14.0 14.5 15.0
Wave Length in Microns
COMPOUND
HCl, SbH₃, NH₃
SOURCE AND PURITY
STATE VAPOR
TEMPERATURE ~25°C
CELL LENGTH
LABORATORY
NAVAL RESEARCH LABORATORY

FIG. 9. Some absorption bands of hydrogen chloride, stibine, and ammonia; NaCl prism.

corresponding to about f:1. Third, the use of curved slits, large prisms, Wadsworth-Littrow mounting giving maximum resolving power, and high-quality optics, was adopted and careful adjustment of all optical parts made to provide good definition. Actually, the off-axis parabolic mirror, even after refiguring, is not as perfect as the best off-axis mirrors made by cutting them out of a large on-axis parabolic mirror. Substitution of such a mirror for that used now should improve the performance of the spectrometer, especially at short wave-lengths. Fourth, precision machine work was insisted on throughout in order to attain the full accuracy of measurement which the optical system is capable of yielding. Finally, the location of the controls, the automatic prism changing, the wave-length indicator, etc., were designed to make the operation of the instrument as convenient as possible.

Although it is not intended to discuss methods of recording here, it is believed that the method described, e.g., recording of energy signal, is superior for the purpose of illustrating the performance of a spectrometer. Other more convenient methods of recording—such as direct recording of transmission—can also be used with the present instrument.

The writers wish to acknowledge their indebtedness to Professor William Schriever, Mr. Elmer C. Miller, and Mr. Ralph E. Fearnow for advice and help with the design of the spectrometer. Special thanks are due Mr. Fearnow for the care and skill with which he constructed the instrument.

Some New Aspects of Germanate and Fluoride Phosphors*

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RCA *Laboratories, Princeton, New Jersey*

By crystallizing with excess magnesium oxide the photo-luminescent efficiency of manganese-activated magnesium orthogermanate has been increased 500 percent. The emission spectra of these materials extend from 6100A to 6700A, and exhibit a unique fine structure consisting of four bands about 50A wide and having peaks approximately 90A apart. Small changes in composition leave the positions of the emission peaks invariant but greatly change the relative intensities of the peaks. This fine structure offers a valuable key to determining the optical transitions involved in the luminescence of manganese-activated phosphors. Manganese-activated zinc-magnesium fluoride phosphors exhibit under cathode-ray excitation the longest truly-exponential afterglow of any efficient known luminescent material. Crystallization of these phosphors under 100 atmospheres of hydrogen fluoride does not greatly alter their luminescent characteristics. Fluoride phosphors, particularly manganese-activated zinc fluoride, possess the unique feature among luminescent materials of being capable of evaporation and condensation in a vacuum to form thin uniform transparent layers without unduly affecting the luminescent efficiency.

I. GERMANATE PHOSPHORS

EFFICIENT luminescent solids, such as copper- and silver-activated zinc-cadmium sulfides and manganese-activated phosphors in general have broad and featureless emission spectra. Cooling to liquid air temperature fails to reveal any fine structure in the emission spectra of these materials.[1] In a program of research on the manganese-activated magnesium germanate phosphors, a unique fine structure of emission has been discovered.

To demonstrate that the emission spectrum and other properties of this phosphor are not caused by unknown impurities, the method of preparation will be discussed first. Preliminary work on the manganese-activated magnesium germanate phosphors indicated considerable variation in luminescent properties between materials prepared with germanium dioxide from different sources. Spectroscopic analysis proved that the distillation of C. P. germanium dioxide as germanium tetrachloride by a procedure originally used by Müller[2] effectively removes aluminum, chromium, and iron. Measurements of the luminescent efficiencies of phosphors prepared with unpurified and purified germanium dioxide showed purification to be mandatory. The actual phosphor is obtained by adding the calculated volumes of solutions of magnesium nitrate, zinc nitrate, and manganese nitrate to the weighed, powdered, purified germanium dioxide in a platinum crucible; stirring the suspension while evaporating to dryness; and grinding the product to 220 mesh and crystallizing at 1100°C or 1200°C.

The measurements of visual luminescent efficiency of the germanate phosphors were made with a MacBeth Illuminometer. Excitation to emission intensities of about 30 μlamberts with ultraviolet and about one μlambert with α-particle excitation were used. The ultraviolet excitation source consisted of a special fluorescent lamp with filter producing a broad band from 3400A to 3900A and with a peak at 3650A. The α-particle excitation source consisted of a gold foil containing 1200 μgrams of radium/in.^{2}. Measurements of the emission intensities of the more efficient germanate phosphors were also made after 500 μgrams of radium had been precipitated on each gram of phosphor particles. A Cenco Grating Spectrograph with a dispersion of 16A/mm throughout the wave-length range from 2300A to 7000A was used in measuring the emission spectra.

Earlier work on the manganese-activated mag-

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** Now Assistant Professor of Chemistry, University of North Carolina, Chapel Hill, North Carolina.

[1] J. Ewles, Proc. Roy. Soc. London **167A**, 34 (1938).

[2] E. H. Archibald, *The Preparation of Pure Inorganic Substances*. John Wiley and Sons, Inc., New York, 1932, p. 220.

nesium germanate phosphor was confined to the metagermanate, $MgGeO_3$, and the orthogermanate, Mg_2GeO_4.[3] The metagermanate, and, in fact, any magnesium germanate containing more germanium dioxide than that corresponding to ortho proportions, was found to form an inefficiently-photoluminescent material of an apparently glassy structure. The substitution of beryllium or zinc oxide up to 10 mole percent for the magnesium oxide affects only slightly the luminescent efficiency.

The addition of magnesium oxide in excess of ortho proportions was found to produce a large unexpected increase in the luminescent efficiency of the manganese-activated magnesium germanate phosphor excited with ultraviolet excitation. For example, with 3650A excitation the phosphor prepared according to the formula $4MgO \cdot GeO_2 \cdot .01Mn$ has 500 percent the efficiency of the most efficient manganese-activated magnesium orthogermanate; namely $2MgO \cdot GeO_2 \cdot .01Mn$. This phosphor containing twice as much magnesium oxide as the orthogermanate and crystallized at 1100°C has the maximum luminescent efficiency, but the maximum with respect to magnesium oxide content is quite broad. Phosphors prepared according to the formulae $3MgO \cdot GeO_2 \cdot .01Mn$ and $5MgO \cdot GeO_2 \cdot .01Mn$ have about 75 percent the efficiency of the optimum composition. Even as great an excess of magnesium oxide as that corresponding to the formula $10MgO \cdot GeO_2 \cdot .01Mn$ results in a phosphor with the same luminescent efficiency with ultraviolet excitation as the original orthogermanate.

A possible explanation of the advantage of the excess magnesium oxide over ortho proportions is that the germanium ion being larger than the silicon ion and the magnesium ion being smaller than the zinc ion, more magnesium can be present in the germanate lattice than corresponds to ortho proportions. In accord with this interpretation, it is found that manganese-activated magnesium orthosilicate, zinc orthosilicate, and zinc orthogermanate phosphors do not show improved luminescent efficiencies on crystallizing with excess metal oxide. X-ray diffraction powder photographs indicate that neither the structure nor the lattice constants of the magnesium orthogermanate are appreciably altered on crystallizing with excess magnesium oxide. With electron or α-particle excitation, however, the luminescent efficiency of the magnesium germanate phosphor is not as pronounced a function of magnesium oxide content as with ultraviolet excitation.

Figure 1 shows the emission spectrum with 3650A excitation of the most efficient manganese-activated magnesium germanate phosphor. The points were taken from a densitometer curve made from the grating spectrogram. As is clear from this figure the emission spectrum of the manganese-activated magnesium germanate phosphor[4] exhibits an unusual fine structure consisting of four peaks 210 ± 10 cm^{-1} apart and each about 0.02-electron volt wide. Magnesium germanate is the only known manganese-activated phosphor that has such a fine structure. The widths of the individual bands, unlike the usual broad and featureless emission bands, can be accounted for by the coupling of the excited electron to the lattice vibrations. The substitution of zinc in part for magnesium and changes in the ratio of metal oxides to germanium oxide leave the positions of the individual peaks invariant but greatly change the relative intensities of the peaks. This effect is shown in Figs. 2–4. By selecting a composition that accents the short wave-length peaks, the visual luminescent efficiency is increased.

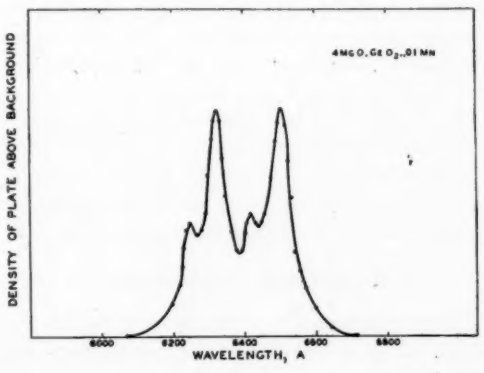

FIG. 1.

[3] H. W. Leverenz, U. S. Patent No. 2,066,044 (December 29, 1936).

[4] Commander T. Davies of the U.S. Navy and members of the National Bureau of Standards first observed that the emission spectrum of this material consists of more than one band.

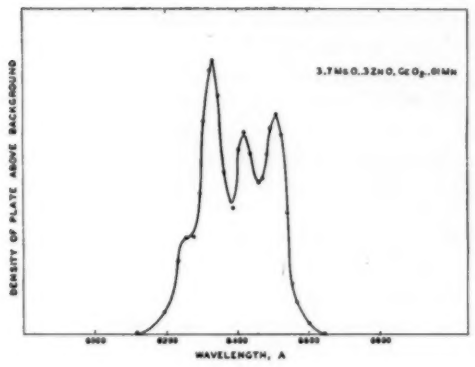

FIG. 2.

Bethe[5] has calculated the effect of crystal fields of different symmetries on electronic energy levels. Splitting of the levels occurs. The number of sublevels and the magnitude of the splitting depend on the quantum character of the level and on the symmetry of the crystal field. Freed and Weissman[6] have obtained qualitative confirmation of this theory applied to rare earth crystals. In that Bethe predicts splitting of the order of 250 cm^{-1}, the fine structure of the emission spectrum of the manganese-activated magnesium germanate probably results from this effect. An alternative but related explanation is that the individual peaks are due to forbidden transitions characteristic of the activator, but the transition probabilities are increased by the symmetry of the crystal field about the activator being destroyed by the disorder produced by the high temperature crystallization, by the excess magnesium oxide, and by the substitution of zinc oxide. A quantitative theoretical analysis of the fine structure according to both interpretations should indicate the correct explanation and provide a valuable clue to the precise nature of the transitions involved in the luminescence of manganese-activated phosphors in general.

II. FLUORIDE PHOSPHORS

The manganese-activated zinc-magnesium fluorides[7] have with cathode-ray excitation the longest, truly-exponential afterglow of any of the efficient known phosphors. Except for a small anomalous region of composition, the afterglow of the phosphors in this system persists over three orders of magnitude of emission intensity before appreciable departure from exponential phosphorescence occurs. The initial afterglow is practically independent of temperature indicating the spontaneous character of the luminescent process. The average lifetime of the excited state is $0.083 \pm .005$ second for the manganese-activated zinc fluoride and is $0.092 \pm .005$ second for the manganese-activated magnesium fluoride.

Ordinarily, these phosphors are prepared from the metal oxides, including about one mole percent manganese, and excess concentrated hydrofluoric acid in platinum apparatus and after drying are heated at 700 to 900°C. Also, they may be prepared from solutions of the metal sulfates and potassium fluoride. These methods of preparing fluorides result by hydrolysis in the presence of small indeterminate amounts of the oxide and the oxyfluoride. Subsequent heating at the crystallization temperature in a furnace with a doubtfully-neutral atmosphere may lead to further oxidation. It is clear, therefore, that the manganese-activated fluoride phosphors may contain metal oxides and oxyfluorides that affect the luminescent characteristics.

One method of minimizing the oxide and oxyfluoride content and also of possibly varying the crystal faults in fluorides is to crystallize the phosphor in an atmosphere of anhydrous hydrogen fluoride. A gold-lined stainless steel pressure bomb was designed and constructed for this work. The procedure used is as follows: After cooling the cylinder with dry ice, sufficient anhydrous hydrogen fluoride to produce 100 atmospheres at the crystallization temperature is poured into a platinum crucible in the bottom of the bomb; the manganese-activated zinc fluoride as a loose powder in an open platinum dish is inserted in a framework in the center of the cell; and the bomb is assembled, sealed with a nickel gasket. It is then lowered into a pot furnace at 200°C and heated to the crystalliza-

[5] H. Bethe, Ann. d. Physik **3**, 133 (1929).
[6] S. Freed and S. I. Weissman, J. Chem. Phys. **8**, 878 (1940).
[7] As early as 1930 M.C. Banca of the RCA prepared phosphors of this composition; the Geophysical Laboratory later rediscovered these phosphors, and their useful luminescent characteristics were recognized; subsequently R. H. Plumlee of the RCA developed improved methods of preparation.

tion temperature in a half hour. After being at the crystallization temperature for a half hour, the bomb is removed, allowed to cool to room temperature, and opened.

Results obtained with this procedure indicate that the manganese-activated zinc fluoride phosphor crystallized at 600°C or 700°C in 100 atmospheres of anhydrous hydrogen fluoride has about 15 percent greater luminescent efficiency than the same material crystallized in a relatively neutral atmosphere at the same temperature. No detectable effect on the spectral distribution of emission or on the phosphorescence resulted from the presence of 100 atmospheres of anhydrous hydrogen fluoride during crystallization. It is concluded that the low concentration of oxides and oxyfluorides in fluoride phosphors merely decreases the luminescent efficiency somewhat, without appreciably affecting the other luminescent characteristics.

A rather interesting special technique of fluoride phosphor preparation has been found to be the evaporation of either the unfired or the crystallized material in a vacuum and condensing the phosphor to form the actual luminescent screen. Not only would this technique be expected to distill the fluoride from the less volatile oxides, but also a continuous, non-grainy screen, transparent in thin layers, is formed. The improved resolution of a non-grainy screen and the improved contrast of a transparent luminescent screen are generally recognized. The conventional cathode-ray tube screen consisting of particles 3 to 15μ in diameter in a deposit 5 to 50μ thick is obviously a crude device to be excited by 5 kv electrons that penetrate only 0.5 to 1.0μ[8] into the phosphor crystals. As well as possessing improved contrast and resolution character, the evaporated luminescent screen has the additional advantages of greater efficiency due to little loss of emitted light as the screen can be deposited 0.5 μ thick or whatever thickness is optimum for the operating conditions and increased thermal and electrical conductivity because of the continuous nature of the deposit.

One of the devices in which fluoride test screens were prepared by evaporation is shown in Fig. 5. This apparatus was designed to permit variations in the temperature of the surface on which the phosphor is condensed. The procedure used is as follows: The phosphor powder is first pressed at 2000 lb/in.^{2} in a hydraulic press to form a pellet which does not spatter on heating in a vacuum. The Pyrex tube shown in Fig. 5 is cracked open and the pellet in a small platinum crucible and the control standard are inserted. The apparatus is then sealed together, sealed on a pumping system, and baked in an oven at 375°C for three hours, simultaneously evacuating to a pressure of 2×10^{-5} mm-mercury or less. The baking is necessary to desorb the water vapor which would otherwise react with the condensing phosphor. After cooling the apparatus to room temperature, the constant temperature bath is filled and, when thermal equilibrium is established, the phosphor is evaporated using a radiofrequency oscillator and coil. The rate of evaporation is adjusted so that the pressure as read on an ioniza-

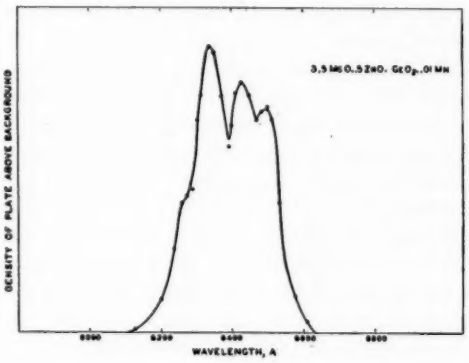

FIG. 3.

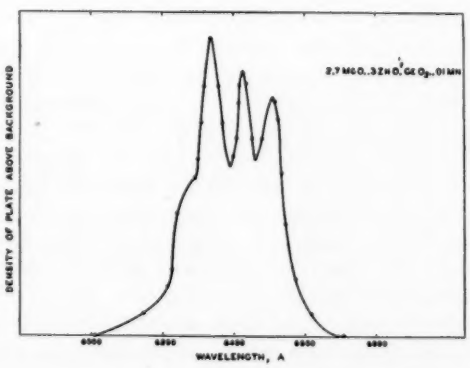

FIG. 4.

[8] H. W. Leverenz, J. Opt. Soc. Am. 27, 23 (1937).

tion gauge does not exceed 10^{-4} mm-mercury. The control standard at room temperature, as well as the base of the constant temperature bath, is coated with a deposit of the luminescent material. The thickness of the deposit can be estimated by counting interference fringes. When the evaporation is complete, the constant temperature bath is emptied, and the luminescence of the deposit is examined visually by allowing the pressure to rise and exciting a gaseous discharge which excites the phosphor. Finally, the tube is cracked open; the polished Pyrex surface cracked off; and the deposit on this surface and on the control standard are quantitatively studied in the spectroradiometer.

In the case of the manganese-activated zinc fluoride phosphor, the luminescent efficiency of the condensate was found to be a function of the temperature of the surface on which condensation occurs, of the thickness of the deposit, and of the rate of evaporation. Evaporation on polished Pyrex surfaces at from 25°C to 100°C produced the most efficient and homogeneous deposits. The deposit produced on a surface in contact with a dry ice-acetone mixture measured only 16 percent the luminescent efficiency of the control deposit at room temperature, and a deposit produced on a surface in contact with liquid air measured only 6 percent. In addition, the latter deposit consisted of a mosaic of crystals loosely hanging vertically from the surface. At the low temperatures, the impinging molecules of zinc fluoride and manganese fluoride apparently do not have sufficient mobility to form a lattice of the type conducive to high luminescent efficiency. Luminescent screens formed by evaporating zinc fluoride containing 0.6 mole percent manganese fluoride and condensing on polished Pyrex in deposits as thin as 0.2μ have a visual efficiency of 8 percent that of standard Willemite. The same material evaporated 3μ thick measured 20 percent. Thicker deposits were formed with 45 percent the visual efficiency of Willemite. One screen, evaporated at a particularly favorable rate, was scraped from the glass surface and tested in the usual thick patch. This sample measured 105 percent the visual efficiency of standard Willemite. The efficiency measurements were made with excitation by electrons at 6 kv and 2μ amp./cm.^{2}. The thicknesses of the deposits were determined by weighing the polished Pyrex screen bases on a microbalance before and after evaporation.

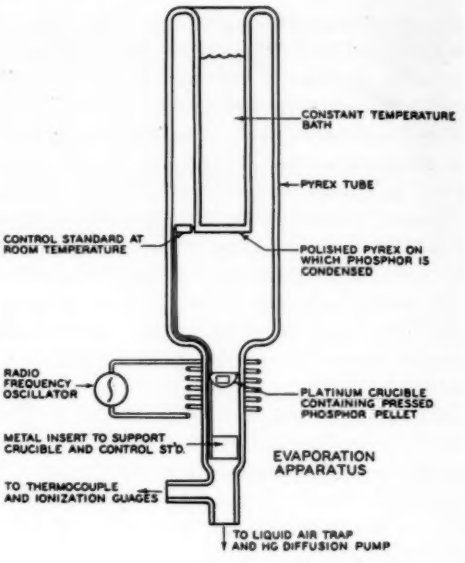
CONSTANT TEMPERATURE BATH
PYREX TUBE
CONTROL STANDARD AT ROOM TEMPERATURE
POLISHED PYREX ON WHICH PHOSPHOR IS CONDENSED
RADIO FREQUENCY OSCILLATOR
PLATINUM CRUCIBLE CONTAINING PRESSED PHOSPHOR PELLET
METAL INSERT TO SUPPORT CRUCIBLE AND CONTROL ST'D.
EVAPORATION APPARATUS
TO THERMOCOUPLE AND IONIZATION GUAGES
TO LIQUID AIR TRAP AND HG DIFFUSION PUMP

FIG. 5.

The above results demonstrate the unique property of fluoride phosphors, manganese-activated zinc fluoride in particular, of being capable of evaporation and condensation in a vacuum without loss of high luminescent efficiency. Zinc fluoride and manganese fluoride have practically the same melting points and vapor pressures at the evaporation temperature so that from a common crucible the two constituents distill off in about the same proportions as in the original phosphor. Magnesium fluoride and manganese fluoride have widely different melting points and vapor pressures so that in order to prepare an evaporated manganese-activated magnesium fluoride phosphor, the constituents must be evaporated from different crucibles maintained at different temperatures.

Independent of the technique of preparation, the fluoride phosphors have one disadvantage; namely, a tendency to burn or lose their efficiency under high current densities. The mechanism of this loss of efficiency is as yet unknown. In the case of the thin evaporated deposits, less than 0.5μ thick, this tendency to lose efficiency under electron excitation is increased. In fact, from

measurements made on the manganese-activated zinc fluoride phosphor, the rate of loss of efficiency with constant electron excitation seems to be approximately an inverse function of screen thickness.

Attempts were made to evaporate other phosphors without loss of luminescent efficiency. Of the aluminate, borate, germanate, oxide, selenide, silicate, sulfide, tungstate, and vanadate phosphors, only the sulfides and selenides evaporated without decomposition. A demountable bell jar vacuum system, complete with an electron gun to test the deposit at various stages, was used in the separate and simultaneous evaporation of the constituents of sulfide type phosphors. It was observed that incident zinc sulfide molecules are completely reflected by surfaces at 650°C. Deposits of zinc sulfide, zinc sulfoselenide, and zinc-cadmium sulfide condensed on surfaces below this temperature failed to show appreciable luminescent efficiency. De Boer[9] has reported sulfide phosphor screens prepared by the evaporation technique. The author found that subsequently heating the deposit resulted in improved luminescent efficiency. The luminescent characteristics of the resulting phosphor are strongly dependent on the atmosphere in which the material is activated. For example, the zinc oxide phosphor, usually prepared by heating zinc oxide in a reducing atmosphere, can be prepared in thin layers by evaporating zinc sulfide and subsequently oxidizing and activating in an oxygen atmosphere at about 600°C.

Summarizing, it can be said that uniform, transparent, non-grainy luminescent screens of zinc oxide, zinc-cadmium sulfide, and zinc sulfoselenide with about 5 percent the visual efficiency of standard Willemite can be prepared by evaporating the constituents separately and simultaneously, and subsequently heating the sublimate in a suitable atmosphere. The fluoride phosphors, however, are the only known luminescent solids that evaporate preserving luminescent efficiencies of the order of standard Willemite and can be excited to luminescence without activation after condensation.

[9] J. H. DeBoer, U.S. Patent 1,954,691 (April 10, 1934).

III. ACKNOWLEDGMENTS

The author is indebted to the following members of the RCA Laboratories: Dr. R. E. Shrader for advice on both the germanate and fluoride research; Mr. S. Lasof for taking the grating spectrograms; and Drs. J. Ruedy and F. H. Nicoll for advice and assistance on the evaporation work. In addition, the author is indebted to Mr. C. W. Wallhausen of the U. S. Radium Corporation for precipitating the radium salt on the germanate phosphors. Finally, the author would like to acknowledge the valuable discussions with Professor H. Eyring of Princeton University on possible explanations of the fine structure of emission of germanate phosphors.

Optical Aids for Subnormal Vision

THE Committee on Sensory Devices of the National Research Council has available a limited number of reprints of a condensation of a "Report on a Survey of Optical Aids for Subnormal Vision," by V. S. Ellerbrock, recently published in the Journal of the Optical Society of America. There are also available a limited number of copies of the original unabridged report. This report, which was prepared to guide the Committee in formulating a program of research, deals with the theory and practical aspects of telescopic spectacles, loupes, reading-glass magnifiers, and projection systems, as well as with recommendations for possible improvements in these devices. Copies of either of these reports will be mailed to professionally interested persons on request to the:

Dartmouth Eye Institute
4 Webster Avenue
Hanover, New Hampshire

Letters to the Editor

Note on the Stability of Phosphors under Ultraviolet Irradiation

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January 30, 1947

EXPERIMENTS on the photo-decomposition of phosphors, which Meister and Nagy recently reported in this journal,[1] led these authors to conclude that direct, primary photolysis took place to a marked degree when phosphors were exposed to ultraviolet radiation of wavelengths shorter than 2000A. Their findings were in contrast to observations and conclusions which the writer had reported[2] for certain phosphors and for such energies as are encountered in F lamp operation.

Unfortunately, the experiments carried out by these authors were not performed in the complete absence of free or adsorbed oxygen and oxygen compounds. However, their absence is a condition sine qua non in studies of phosphor stability with radiation which can photo-decompose these oxygen compounds. Thus the writer believes that Meister and Nagy's results are not at variance with his own conclusions because they probably did not observe primary photolysis, but secondary chemical decomposition of the phosphors, occasioned by the action of photochemically produced active oxygen on the phosphor activators. Phosphors containing multivalent elements are usually very sensitive to attack by chemically active oxygen, whether in atomic or excited form or as ozone, NO_2, etc. The attack is manifest by a loss of fluorescent brightness, change of potential, and several other features.

Active oxygen can be produced from molecular O_2, CO_2, etc. through absorption of radiation shorter than about 2000A. In this case, low pressure Hg quartz lamps were used which produce a considerable amount of λ1849 radiation. The "Sterilamps" used by the authors produced also some λ1849 radiation whose intensity diminishes rapidly during the burning of the lamps. Assuming active oxygen to be responsible for the phosphor deterioration, one should expect exactly the type of depreciation curves which were observed. The Sterilamp curves should become flat at an earlier reading and at a higher level, while the quartz lamp curves should be steeper and should continue to decline even at lower levels. The difference in results under quartz and Sterilamp irradiation appears to be one of degree rather than principle.

The writer concludes that the authors had oxygen etc. present in their experiments because the phosphors were apparently not adequately baked in vacuum immediately prior to seal-off and irradiation. This, however, is a crucial point as is well known from the manufacture of fluorescent lamps or cathode-ray tubes. In our experience, unbaked phosphors as commonly handled retain of the order of 100 micron-liters and more of adsorbed gases per gram substance, enough to cause the observed brightness loss when active oxygen is produced by photolysis.

Phosphor depreciation was also observed in one experiment in which the phosphors were baked during exhaust. Here, however, the test conditions (*packing* of the phosphors into a tube, baking for only ½ hour) were such that complete removal of volatilizable impurities was not likely. Calculations on pumping speed as well as an actual experiment, inspired by the paper, showed that the baking time was short by a factor of at least 100, if, indeed, it is at all possible to produce a good vacuum in a reasonably short time in large masses of closely packed powder particles of micron size. Their resistance to flow of gases is extremely high.

If photolysis of phosphors (silicates, tungstates) should occur under irradiation with 2–20 milliwatts/cm² of short ultraviolet, it should be observed to an even greater extent in lamps filled and operated with the rare gases alone, without any mercury. Common experience, however, indicates greater phosphor stability in lamps filled with low-pressure argon or neon.

In one of their conclusions, the authors allow that oxygen may be involved in phosphor depreciation under irradiation with λ1849. The writer believes that this was the chief cause of the brightness losses observed. A repetition of the experiments under refined experimental conditions seems highly desirable.

[1] G. Meister and R. Nagy, J. Opt. Soc. Am. **36**, 696 (1946).
[2] H. C. Froelich, Trans. Am. Electrochem. Soc. **87**, 365 (1945); J. App. Phys. **17**, 573 (1946).

Note on the Number of Distinct Chromaticities

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February 3, 1947

AN estimate of the number of distinct, real, equiluminous colors has been published by M. A. Biot.[1] Using the UCS diagram of Judd,[2] Biot computed that 1800 distinct colors are represented by the region of spectral chromaticities, bounded by the spectrum locus and the straight lines between the extremities of the spectrum locus and the point representing a blackbody at 4800°K. Biot points out that the remaining real (non-spectral) region of Judd's UCS diagram has an area about twice as great as the spectral region, implying a total of about 5500 distinct, real colors. Biot made an independent determination of the number of distinct chromaticities along the straight line joining the extremities of the spectrum locus. He compared his result (20) with the number of distinct colors in the spectrum (130) indicated by Judd's Fig. 2. He assumed the same average number of distinct colors between white and the various purples of maximum purity as Judd's Fig. 9 indicates between white and various spectrum colors, and concluded that there are only about 300 distinct non-

spectral chromaticities, giving a total of 2100 distinct real chromaticities.

Silberstein[3] computed that a truly uniform-discrimination map of the chromaticity diagram would have an area of about 13×10^4, in terms of a unit of length representing the standard deviation of chromaticity-matching. MacAdam[4] found that a chromaticity difference three times as great as a standard deviation is just noticeable. The maximum number of chromaticities, each just noticeably different from its nearest neighbor, would be represented on the surface specified by Silberstein at the corners of a network of nearly equilateral triangles with sides equivalent to approximately three standard deviations. Occasional irregularities in the array of triangles would be caused by the intrinsic curvature of the uniform-discrimination surface.[3,5] In such an array there are twice as many triangles as corners. Therefore, the area of the uniform-discrimination surface corresponding to each distinct color is $(3^{3/2})/2$, or 7.8, in terms of the unit of length based on the standard deviation. This indicates that about $(13/7.8)\times10^4$ or about 17,000 distinct colors of equal luminance are possible, under the conditions of observation used by MacAdam and Nutting:

Matching Field	—15 millilamberts 2 degrees diameter, bipartite fine dividing line on vertical diameter.
Surround	—7.5 millilamberts daylight quality 42 degrees diameter concentric with field immediately contiguous to field.
Observation	—normal color vision monocular 2.6 millimeters diameter artificial pupil.

The area of the regions representing non-spectral chromaticities in the ICI coordinate system is $0.07(x^2)$. In that region the average value of g^3 (the ratio of a small element of area on the uniform-discrimination surface to the corresponding element of area on the ICI chromaticity diagram) is 6×10^5. The area of the non-spectral region on the uniform-discrimination surface is therefore about 4.2×10^4, or about one-half the area representing spectral colors. This may be compared to Judd's UCS diagram in which the non-spectral region has an area twice as great as the spectral region. On the other hand, it may be compared with Biot's estimate that there are only one-sixth as many distinct non-spectral colors as spectral.

Biot's conclusion may be questioned because the numbers of distinct spectrum and pure non-spectrum chromaticities, which he compares, were not determined under equivalent conditions of observation. Figures 20 and 21 of MacAdam's paper[4] indicate about 750 standard deviation units in the spectrum, or about 250 just-noticeable differences. Figure 22 of that paper indicates (for the same observer and conditions) about 420 standard deviation units along the line connecting the extremities of the spectrum locus, or 140 just-noticeable differences. This indicates that there are at least half as many distinct chromaticities in the series of purest non-spectral colors as there are in the spectrum.

[1] M. A. Biot, Annales de la Société Scientifique de Bruxelles **60**, 149 (1946).
[2] D. B. Judd, J. Opt. Soc. Am. **25**, 24 (1935).
[3] L. Silberstein, J. Opt. Soc. Am. **33**, 1 (1943).
[4] D. L. MacAdam, J. Opt. Soc. Am. **32**, 247 (1942).
[5] D. L. MacAdam, J. Frank. Inst. **238**, 195 (1944).

Criterion for Resolution

PAUL C. CROSS AND EUGENE R. NIXON
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January 24, 1947

IN a recent article[1] the authors proposed (and gave a detailed application to the case of an absorbing prism) a criterion of resolution differing from the Rayleigh criterion in that it determined the true limit of resolution. It has been kindly called to our attention by C. Candler of the Royal Institution that an equivalent criterion was discussed by C. M. Sparrow.[2] We wish to acknowledge here Sparrow's previous treatment and to call attention to the fact that he discussed the application of this criterion to several optical systems. He also reported experimental results demonstrating the validity of the criterion for a variety of conditions.

[1] P. C. Cross and E. R. Nixon, J. Opt. Soc. Am. **34**, 517 (1944).
[2] C. M. Sparrow, Astrophys. J. **44**, 76 (1916).

Book Review

Definition and Measurement of Gloss

V. G. W. HARRISON. Pp. vi+145, Figs. 28. The Printing and Allied Trades Association (PATRA), Charter House Square, London E. C. 1, England, 1945. Price 10/-net.

This is believed to be the first book ever to appear on the subject of gloss. It is designated merely a survey of published literature, but the author has added much comment and observation of his own to round out an excellent study of the whole field of gloss recognition and evaluation. The author is in the employ of PATRA, which has published this monograph. However, the book is in no sense restricted to subject matter of interest to the paper and printing trades.

The monograph is divided into four parts: (1) Physical factors influencing our sensations of gloss or luster, (2) Physiological and psychological factors influencing our sensations of luster, (3) Attempts at practical gloss measurement, and (4) Discussion of our present state of knowledge.

The problem of measuring gloss is seen to be primarily

psychophysical; it is to find what scale or scales will give physical values correlating with everyday visual estimates of gloss and luster. At present there exists no general agreement on what constitutes a proper physical procedure for measuring gloss, nor is it likely that a single standard method can ever be made to serve all cases. This follows from the fact that the physical factors affecting visual judgments of glossiness are found to vary markedly from one material to another, and from one observing situation to another. Add to this the fact that psychologists have reached no agreement on definitions of the gloss sensation, and one can see that the whole subject is still in a confused condition.

The reviewer joins with Dr. Harrison in placing some of the blame for this confusion on oversimplification of the problem contained in the traditional pictures of specular and diffuse reflectance. As Dr. Harrison says it: ". . . In the popular accounts of gloss one is constantly finding the statement that cases of partial gloss are simple combinations of diffuse and specular reflection, and the two may be isolated and measured. Would it were so easy!" Only where surfaces are smooth, it is pointed out, can specular reflectance be measured with precision. Of diffusion it is said, ". . . Although quite a number of surfaces obey Lambert's law approximately under certain conditions of illumination and viewing, no material is known for which it holds exactly; and, furthermore, none of the several attempts to provide a theoretical basis for it have been successful. . . . The value of Lambert's law lies in its convenience and simplicity . . . ."

Of goniophotometric curves, which have been offered by some as the cure-all for the gloss-measurement problem, Dr. Harrison says ". . . There is no difficulty about getting such curves to any reasonable degree of accuracy, though the determination is rather tedious; the real difficulty lies in knowing what to do with the curves when one has them."

In spite of these misgivings, the author feels that the gloss-measurement problem can probably be solved by the development of perhaps three or four standard scales of measurement. He indicates that PATRA is embarking on a program of research aimed at finding the best gloss scales at least for the paper and printing industries.

Although one or two of the author's admitted imperfections were noted in passing, none seemed serious. The little book is strongly recommended for inclusion in the library of every worker concerned with evaluating appearance in a scientific manner. It is by far the most comprehensive treatment of the gloss problem the reviewer has seen to date.

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Henry A. Gardner Laboratory, Inc.

Photographic Abstracts

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